1 Introduction

During the financial crisis and its aftermath, those segments of the economy most exposed to the accumulation of mortgage debt have tended to fare the worst. Whether it is by industry (construction), by geography (sand states), or by household (the most indebted), the presence of greater mortgage debt has led to weaker economic outcomes (see for example, Mian and Sufi, 2009, and Dynan, 2012). Moreover, research suggests that financial crises accompanied by a housing collapse may be more severe and be associated with slower recoveries (Reinhart and Rogoff, 2009, Howard, Martin, and Wilson, 2011, and International Monetary Fund, 2012).

These observations lead to an apparently natural macroeconomic policy prescription: restoring stronger economic growth requires reducing accumulated mortgage debt. In this paper, we consider this proposal in an environment where debt is indeed potentially damaging to the macroeconomy, but where the government and private sector have a range of possible policy interventions. We show that while debt reduction can support economic recovery, other interventions can be more efficient, and whether the debt reduction is done by the government or by lenders matters for its efficacy and desirability. Hence, while the intuitive appeal

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of debt reduction is clear, its policy efficiency is not always clear, and the argument is more nuanced than
the simple intuition.

We begin with a simple environment with homeowners, lenders, and a government. We start from the sim-
plest case with perfect information where all households are liquidity constrained. We then layer on default,
private information, heterogeneous default costs, endogenous provision of private mortgage modifications by
lenders, and an equilibrium house price response.

Initially, homeowners may consider defaulting on their mortgages because they are liquidity constrained
(cash flow constrained) or because their mortgage exceeds the value of the home (strategic default), or both
considerations may be present. The government has finite resources and maximizes utility in the planner’s
problem. We initially consider a two-period model with exogenous home prices and then allow for general
equilibrium feedback. We ask, “What type of intervention is most effective? - taking into account the
government budget constraint and the program’s effectiveness at supporting the economy?” We consider a
general class of interventions that includes mortgage modifications, such as interest rate reductions, payment
deferral, and term extensions, as well as mortgage refinancing and debt write-downs. We extend the model
to include default, with known, uncertain, and unobserved default costs, dynamic default timing, and lender
renegotiation.

The model is abstract and simple by design, to focus on only the minimum characteristics necessary
to highlight these mechanisms in the housing market. It omits many interesting and potentially relevant
features of the housing market and of the economy more generally. For example, we generate a "crisis period"
exogenously by specifying lower income in one period to disrupt consumption smoothing by households. We
could, in principle, embed our housing model in a general equilibrium framework that would derive lower
income and generate the scope for housing policy endogenously, as in Eggertson-Krugman (2010), Hall
(2010), Guerrieri-Lorenzoni (2011), and others. In the former, for example, nominal values of debt and
sticky prices, along with the liquidity constrained households which we include, causes output to be demand
determined; hence there is scope for policy to improve macroeconomic outcomes when the debt constraint
binds and the nominal interest rate is zero. Including our model in such a structure would also allow
examination of how housing policy feeds back from the housing market to the macroeconomy. While this
is an interesting route to pursue, given our focus on distinguishing between various types of housing market
interventions, the additional impact that may come from the macroeconomic feedback is scope for further work.

Here the crisis period is defined by low income, which constrains consumption due to liquidity constraints. The household cannot borrow against future income nor against housing equity in order to smooth through the crisis. The government has a range of possible policy interventions and a limited budget; we focus on policies related to housing modifications given the severity of the constraints and defaults experienced there. For simplicity, we begin with a case without default to highlight the consumption smoothing issue and the need for transfers that focus on liquidity constrained households during the crisis period. We then add the potential for default and show that optimal policies that concentrate transfers early in the crisis but require repayment later may exacerbate the default problem. These results suggest that payment deferral policies alone (which grant short-term reductions in home payments but are repaid with higher loan balances later), may generate payments that rise too quickly and generate defaults, so payment forgiveness may optimally replace or augment payment deferrals.

We also allow for default to occur for strategic reasons when the value of the loan exceeds the value of the collateral property, or the loan is "underwater". Even in this case, households may not default because of both pecuniary and non-pecuniary costs. Allowing for these deadweight costs, we model the default decision and show that with liquidity constraints, payment reduction focused in the crisis period is still the optimal policy and remains so until the liquidity constraint has been fully relaxed. This result still holds when we allow households to choose the timing of default, as it is optimal for households to delay default when there is an option value of waiting.

When lenders can renegotiate the loan, they also tend to delay in order to preserve the option value of waiting. Without liquidity constraints, lenders concerned about strategic default would optimally offer a debt reduction at end of the period (defined as just prior to default) in order to preserve option value but avoid costly default. We also show that "debt overhang" concerns, that is, the possibility that debt inhibits access to credit and reduces consumption, does not change our results. Even if loan modifications reduce debt overhang, liquidity constraints can be directly and more efficiently addressed by front-loaded policy interventions, rather than through a reduction in contracted debt.

Summarizing, our analysis of loan modifications produces two broad results. First, with liquidity con-
straints, transfers to households during the crisis period weakly dominate transfers at later dates and hence are a more effective use of government resources. These initial transfers could include temporary payment reductions, such as interest rate reductions, payment deferral, or term extensions. This result is robust to including default, various forms of deadweight costs of default, debt overhang, and the easing of credit constraints through principal reduction. Generally, any policy that transfers resources later can be replicated by an initial transfer of resources, but the converse is not true. Second, principal reductions offered by the private sector can reduce any deadweight costs due to strategic default. This conclusion is independent of whether or not liquidity constraints are present. With the potential for delay, these defaults, and hence the debt writedowns would occur just prior to default.

Allowing for endogenous price determination in the housing market reinforces these results. We embed the consumption and policy choice problem in an equilibrium model of housing, with rental housing demand augmented by those households who default on their mortgages and move from home ownership to rental. They key result from this section is that foreclosures by liquidity-constrained households undermine demand and hence prices more than do strategic defaults. Essentially, these liquidity constrained households carry their constraint into the rental market, which constrains their demand and puts further downward pressure on home prices. Strategic defaulters, on the other hand, also incur the deadweight cost of default, but are not in distress and hence have greater demand for housing than do the liquidity constrained. For a policy-maker concerned about foreclosure externalities and home prices, distressed foreclosures by liquidity constrained households are more damaging.

Our results demonstrate that different types of ex post interventions in home lending solve conceptually distinct problems. Payment-reducing modifications, which steepen the profile of payments through payment deferrals, temporary interest rate reductions, or term extensions, for example, address cash flow and liquidity constraints. Reducing loan principal, because it back-loads payment reduction, is inefficient at addressing cash flow issues, but is effective at addressing later period (but not initial) strategic default risk faced by lenders. Interestingly, mortgage refinancing to reduce the borrowing rate also addresses strategic default risk.

These results on ex post modifications are suggestive of the ex ante properties of loan contracts that would ameliorate the problems that arise during a crisis with both borrowing constraints and declining home prices.
Specifically, a contract should allow for disproportionately lower payments when borrowing constraints bind and a reduction in loan obligations when home prices fall to reduce the incentive for strategic default. Such a contract fills the role of automatic stabilizers in the housing market. A "stabilizing" mortgage contract that includes a prepayment option that allows for refinancing into an lower adjustable rate mortgage during the crisis period is consistent with the \textit{ex ante} security design problem, and is near the space of existing contracts.

2 Basic Model

Households derive utility from housing and other consumption goods according to the consumption aggregate, $C_t$, where

$$
C_t = (c_{ht})^{\alpha} (c_t)^{(1-\alpha)},
$$

and $c_{ht}$ is consumption of housing services and $c_t$ is consumption on non-housing goods. The household maximizes linear utility over two periods:

$$
U = C_1 + C_2
$$

We have set the discount factor to one as it plays no role in the analysis.

At date 0, i.e., a date just prior to date 1, the household purchases a home and takes out a mortgage loan. At the date 0 planning date, the household expects to receive income of $\bar{y}$ at both dates. For now, there is no uncertainty. Income is allocated to non-housing consumption and paying interest on a mortgage loan to finance housing consumption. A home of size $c^h$ costs $P_0$ and is worth $P_2$ at date 2. In the basic model, $P_2$ is non-stochastic.\footnote{Later, we will introduce home price and income uncertainty; for now, we take these as given and known to the household.} The home price $P_0$ satisfies the asset pricing equation,

$$
P_0 = rc^h + re^h + P_2,
$$

where $r$ can be interpreted as the per-period user cost of housing. That is, if an agent purchases a home for $P_0$ and sells it in two periods for $P_2$, the net cost over the two-periods is $2rc^h (= P_0 - P_2)$.

To finance the initial $P_0$ outlay, the household takes on a mortgage loan. A lender provides $P_0$ funds to purchase the house in return for interest payments of $l_1$ and $l_2$ and a principal repayment of $D$. For the
lender to break even, repayments must cover the initial loan:

\[ l_1 + l_2 + D = P_0, \]  

(4)

where we have set the lender’s discount rate to one, as well. Given choices of \((l_1, l_2, D)\), non-housing consumption is,

\[ c_1 = \bar{y} - l_1 \quad \text{and}, \quad c_2 = \bar{y} + P_2 - D - l_2. \]  

(5)

The household chooses \((l_1, l_2, D)\) to maximize (2).

It is straightforward to derive that a consumption-smoothing household maximizes utility by setting,

\[ l_1 = l_2 = \alpha \bar{y} \quad \text{and}, \quad D = P_2. \]  

(6)

That is, interest payments on the housing loan are \(\alpha \bar{y}\), and the principal repayment is made by selling the home for \(P_2\). These choices result in consumption,

\[ c_t^h = \frac{\alpha \bar{y}}{r} \quad \text{and}, \quad c_t = (1 - \alpha)\bar{y}. \]  

(7)

Note that with Cobb-Douglas preferences, the expenditures shares on housing and non-housing consumption are \(\alpha\) and \(1 - \alpha\). Since the effective user cost of housing, \(r\), is constant over both periods, the household equalizes consumption over both dates.\(^2\)

### 2.1 Crisis

A “crisis” occurs in the model by allowing an unanticipated negative income shock to hit this household, so that income at date 1 is instead \(y_1 < \bar{y}\), leaving income at date 2 unchanged. There are two ways the household can adjust to this shock. It can default on the mortgage, reduce housing consumption and increase non-housing consumption.\(^3\) Or, it can borrow from date 2, reducing future consumption and increasing current consumption. We first study the second option and assume that the household does not default on his mortgage; we consider default in the next sections. If the household does not default, it will consume too

\(^2\)With linear utility, the consumption allocation is formally indeterminate, but any amount of curvature will produce consumption smoothing in this way.

\(^3\)In principle, the household could sell the home and buy another to reoptimize consumption. We assume that this is costly or not feasible as a way of smoothing consumption for temporary shocks. We discuss borrowing further below.
little of non-housing services at date 1, not just relative to his initial plan but also relative to re-optimized
consumption, $\bar{c}_t$, given actual income at date 1, if the household were able to borrow against future income:

$$\bar{c}_1 = y_1 - \alpha y < \bar{c}_2 = y - \alpha y.$$  \hspace{1cm} (8)

That is, if the household could borrow freely at interest rate of zero, then it would increase date 1 consumption
and reduce date 2 consumption.

A household with other assets, or one with equity in its home, can borrow to achieve this optimum
consumption path. We instead will focus on a liquidity-constrained household. This household has no other
assets, little to no equity on the home, and is unable to borrow against future income. Hence this household
can only adjust its non-housing consumption beyond the liquidity constraint by defaulting on its mortgage,
since it cannot borrow against future income or consume from other assets or home equity.

Let us suppose a government has $Z$ dollars that it can spend to increase household utility. The scope for
government intervention arises in this setting directly because of the liquidity constraint, as in Eggertson-
Krugman (2010), Guerreri-Lorenzoni (2011), and others, and could also be reinforced by an aggregate demand
shortfall, consumption externalities, other credit market frictions, or other considerations. We do not model
these explicitly, as our focus is on the housing market, though we allow for additional considerations in the
next sections. Hence, the government’s budget allocation may result from the government’s intention to
ease liquidity constraints in period 1, or similarly, as a way of implementing countercyclical macroeconomic
policies, since date 1 is the "crisis" period in the model.

Suppose the government chooses transfers $(t_1, t_2)$ in the first and second period, respectively, that satisfy
the budget constraint$^6$:

$$t_1 + t_2 = Z.$$  \hspace{1cm} (9)

$^4$In principle, the household could also sell the home and use the proceeds to buy a new one, reoptimizing over the two types
of consumption. This means that the household is effectively not liquidity constrained, since the home becomes a liquid asset.
We assume that this option is not available to the household, either because transactions costs are high, the home is underwater
and the household has insufficient other assets (so that a home sale - a short sale - requires a loan default), or credit market
frictions that prevent the homeowner from consuming out of real estate wealth.

$^5$The importance of liquidity constraints during the crisis is emphasized in the empirical results of Dynan (2012) using
household consumption data.

$^6$Note that the government’s discount rate is also one, so we do not give the government an advantageous borrowing rate
compared to private agents.
The household’s budget set is now augmented by a transfer in period 1 to help overcome the liquidity constraint, and a second transfer at date 2, resulting in household consumption of

\[ c_1 = y_1 - \alpha \bar{y} + t_1, \quad c_2 = \bar{y} - \alpha \bar{y} + t_2 \]  

(10)

Here we consider only policies related to modifying the mortgage; in the next section, we add default, so that policies are more directly tied to house payments. Various choices of \( t_1 \) and \( t_2 \) can be mapped into standard types of loan modifications. For example, setting \( t_1 > 0 \) and \( t_2 = 0 \) in our notation corresponds to a pure “payment reduction” loan modification which temporarily reduce loan payments, say through a temporary interest rate reduction. A “payment deferral” program offsets initial payment reduction with future payment increases, setting \( t_1 > 0 \) and \( t_2 < 0 \), say through maturity extension or loan forbearance. A program with \( t_1 = t_2 > 0 \), so that payment reductions are equally spread over time, corresponds to a fixed rate loan refinancing (since loan payments are lowered uniformly) and to principal reduction, that is, a reduction in the loan principal that results in reduced interest and principal payments at each date.

The planner maximizes utility,

\[ \max_{t_1, t_2} \left( e_1^h \right)^\alpha (c_1)^{(1-\alpha)} + \left( e_2^h \right)^\alpha (c_2)^{(1-\alpha)} \]  

(11)

subject to (9) and (10). Note that since we are considering the case where the household does not default and hence does not reoptimize housing consumption, the consumption values \( e_1^h, e_2^h = \frac{\alpha \bar{y}}{\theta} \), are invariant to the choice of \( t_1 \) and \( t_2 \).

We can rewrite the planner’s problem as,

\[ \max_{t_1, t_2} v(y_1 - \alpha \bar{y} + t_1) + v((1 - \alpha)\bar{y} + t_2) \quad s.t. \quad t_1 + t_2 = Z \]  

(12)

where,

\[ v(c_t) \equiv \left( \frac{\alpha \bar{y}}{\theta} \right)^\alpha (c_t)^{1-\alpha} \]  

(13)

and we note that \( v(\cdot) \) is concave.

Figure 1 illustrates the solution for non-housing consumption for \( Z = 0 \). The vertical axis graphs \( c_2 \), while the horizontal axis graphs \( c_1 \). The initial point A after the shock has \( c_2 > c_1 \). The red diagonal line traces out the set of points that satisfy the budget constraint, \( t_1 + t_2 = 0 \) (i.e., \( Z = 0 \)) The optimum calls for full consumption smoothing, which is to set \( t_1 > 0 \) and \( t_2 < 0 \) until \( c_1 = c_2 \) (the 45 degree line) at point B.
The figure illustrates a case with $Z = 0$. As $Z$ rises, the red diagonal line shifts outward, but for any given $Z$ we see that payment deferral ($t_1 > 0, t_2 < 0$) is better than payment reduction ($t_1 > 0, t_2 = 0$) because it allows higher transfers in the first period, which is in turn better than principal reduction ($t_1 > 0, t_2 > 0$), where transfers continue beyond the crisis period. This finding is consistent with general results in public finance that transfers into liquidity constrained states enhance utility, since the marginal utility of consumption is high in those states. A reduction in mortgage principal does not transfer liquid assets into those states since the household is by definition liquidity constrained and cannot borrow against his higher wealth. The increase in wealth is implemented by a stream of lower mortgage payments over the life of the loan, which is likely to extend well beyond the crisis period. Hence, gathering those benefits together into a front-loaded transfer is more effective. We highlight this result in this simplest setting because it is robust...
throughout as we add additional features to the model: transfers in the initial crisis period at least weakly dominate policies that transfer resources later.

We have described the solution \((t_1, t_2)\) as the solution the planning problem. However, there is nothing in our setup thus far that precludes the private sector from offering a loan modification. If private lenders could offer contracts with \(t_1 > 0, t_2 < 0\) they would find it profitable to do so. This would correspond to loan refinancing with term extension, for example, which might be desirable to households by reducing payments immediately, but profitable for lenders over the life of the loan. Nonetheless, there are several reasons why policy may still be desirable. While we have not modeled a government’s preference for countercyclical policy, private lenders may not offer the socially optimal amount of modifications if there are credit market frictions, consumption externalities or an aggregate demand shortfall. Hence, it may be optimal for the government to offer or subsidize modifications in addition to available private sector contracts. Moreover, later we will show that with asymmetric information, the market in private contracts may collapse due to adverse selection, which provides further scope for policy intervention.

3 Default at Date 2

In the simple setting above, the optimal solution requires \(t_1 > 0\) and \(t_2 < 0\) so that more consumption can be transferred to the crisis period. That is, it calls for providing the equivalent of a loan to finance date 1 consumption by liquidity constrained households. However, in practice such loans may induce default, by frontloading the benefits and backloading the costs of the program to households. Households, especially households with underwater mortgages, may take the payment deferral and then subsequently default on the loan. In this section, we allow for endogenous default at date 2 to examine how policy interventions at date 1 affect subsequent date 2 default. We later include the opportunity to default at date 1, so that there is a timing element in the default decision.

3.1 The default decision and loan modifications

Suppose that after signing mortgage contracts at date 0, home prices change at date 1 (the price change is unanticipated from the date 0 perspective) and some agents will choose to default, given their income and
the new home price.\textsuperscript{7} For simplicity we initially assume that which households will default is fully known at date 1. There is no further uncertainty between date 1 and date 2, and there is no private information regarding the default propensities of households. Later sections examine the implications of relaxing these assumptions.

We start by analyzing the case where default may occur at date 2, but not at date 1; for now, this is the more interesting case to study since it reduces the incentive to front-load transfers (in contrast to the last section). Formally, we assume date 1 default costs are infinite, which we relax in the next section.

If a household defaults on his mortgage, he loses his home, which was the collateral for the loan, and any equity in the home. Since the household still requires housing services, he then enters the rental market to replace the lost housing services. The household also suffers a default cost, which may represent restricted access to credit markets, benefits of homeownership or neighborhoods, and so on. The date 2 wealth of a non-defaulting household is\textsuperscript{8}

\[ y + P_2 + D + t_2. \]  

(14)

If the household defaults, his wealth instead becomes,

\[ y - \theta \]  

(15)

where \( \theta \) is a deadweight cost of default, and the household may also lose any date 2 home-related transfers.\textsuperscript{9}

As date 2 is an unconstrained date, the household is able to reoptimize consumption decisions given the housing user cost \( r \). It is straightforward to show that utility over date 2 consumption is linear in wealth, either \( y + P_2 - D + t_2 \) or \( y - \theta \), so that the household defaults if,

\[ y - \theta > y + P_2 - D + t_2, \]

so that wealth after defaulting exceeds wealth of continuing to service the mortgage. Thus date 2 utility is given by,

\[ \max[y + (P_2 + t_2 - D), y - \theta] \frac{\psi}{\left(\frac{\alpha}{\gamma}\right)^\alpha (1 - \alpha)^{1 - \alpha}} \]  

(16)

\textsuperscript{7}This begins to introduce home price uncertainty into the model, which we consider more formally later in this section.

\textsuperscript{8}In the deterministic version of the model, the loan is fully collateralized so that \( D = P_2 \), but we carry the more general notation of \( P_2 - D \) through to the case where \( P_2 \) is stochastic.

\textsuperscript{9}This may also be thought of as a tighter liquidity constraint that constrains non-housing consumption or of a loss of non-pecuniary consumption, such as schools or house quality, associated with owning rather than renting housing services.
where \( \psi \) is the marginal value of a dollar at date 2, and will be a constant throughout the analysis.

Define the equity in the home \((P_2 - D)\) plus the default cost as

\[
\phi = P_2 + \theta - D,
\]

which represents the total cost of default to the household. This includes the deadweight cost of default plus any equity in the home, or alternatively, the deadweight cost of default less the liability from the underwater portion of the mortgage. Then the default condition is expressed by the inequality

\[
\phi < -t_2,
\]

which determines whether the household defaults on his mortgage and incurs the deadweight cost of default. Otherwise the household continues to service the mortgage. The variable \( \phi \) determines the slope of the modification program, i.e., \( \phi \) is the maximal repayment or "consumption loan" that the borrower can take without creating the incentive to default in period 2.

We can solve the planner’s problem subject to this default constraint, \( t_2 < -\phi \). The solution will set \( t_2 = -\min(\phi, 0), t_1 = -t_2 + Z \), unless \( \phi \) and \( Z \) are large enough that the household is able to smooth consumption fully using the transfer in period 1 and paying it back in period 2. Otherwise, the default constraint limits the ability to implement the payment deferral policy and the optimal policy becomes less steep. That is, if \( \phi \) is known and is not too large, the optimal policy includes less payment deferral and more payment reduction in the first period, so the optimal policy includes front-loaded payment reduction, and deeper negative equity forces repayment requirements, \( t_2 < 0 \), to be small and potentially negative so as not to induce default. A mechanism such as this is apparent in the observed response of households to crisis-related cash transfers, which as documented by Hsu, Matsa, and Melzer (2014), had a significant effect in reducing foreclosures. They find a large impact of higher unemployment benefits (which are not repaid later) in reducing the probability of default across states and over time.

### 3.2 Loan modifications with heterogeneous default risk

The propensity of homeowners to default may also vary across households, which affects the loan modifications offered. Hence, instead of a fixed value of the default cost, \( \phi \), let us next assume that \( \phi \in [\hat{\phi}, \infty] \) is a random variable realized at date 2. For example, realizations of \( P_2 \) may vary across homeowners, leading
to different realizations of $\phi$. As in previous sections, we continue to assume that at date 0 agents contract under the assumption that home prices are certain. The randomness of home prices is an unanticipated event. The CDF of $\phi$ is denoted as $F(\phi)$. Since borrowers default when $\phi < -t_2$, for given $t_2$ we have that $F(-t_2)$ borrowers default on loans. We will assume the interesting case where $(t_1, t_2)$ are such that it is advantageous for every liquidity constrained borrower to take the modification contract, but a fraction $F(-t_2)$ strategically default on their loans in the second period, when they are unconstrained.

A planner with $Z$ dollars to spend solves,

$$\max_{t_1, t_2} (1 - F(-t_2))E[v(y_1 - \alpha\bar{y} + t_1) + (\bar{y} + t_2 + P_2 - D)\psi|\phi > -t_2] +$$

$$F(-t_2)E[v(y_1 - \alpha\bar{y} + t_1) + (\bar{y} - \theta)\psi|\phi < -t_2]$$

The first line is the utility of the constrained borrowers with high default costs (i.e., high $\phi$) who take the modification and do not default. The second line is the utility of the constrained borrowers who will default.

The government budget constraint requires\(^{10}\)

$$Z - t_2(1 - F(-t_2)) - t_1 = 0$$

A fraction $1 - F(-t_2)$ of borrowers make the repayment of $-t_2$. This repayment plus the $Z$ dollars must cover the initial payment of $t_1$.

Denote $\mu$ as the Lagrange multiplier on the budget constraint. The first order condition with respect to $t_1$ gives,

$$v'(y_1 - \alpha\bar{y} + t_1) = \mu,$$

and with respect to $t_2$ gives

$$(1 - F(-t_2))\psi = \mu ((1 - F(-t_2)) + t_2 f(t_2)).$$

Combining, we find,

$$v'((1 - \alpha)\bar{y} + t_2)\psi^{-1} = 1 + \frac{t_2 f(t_2)}{1 - F(-t_2)}.$$

\(^{10}\)The budget constraint does not require that the program pay for itself unless $Z = 0$. If $Z > 0$, the program provides net funds for mortgage modifications, and date 1 payment reductions can be larger to the extent that they are repaid at date 2 with negative transfers, $t_2 < 0$. 

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The solution is easy to illustrate pictorially. Figure 2 graphs first and second period non-housing consumption for various values of government transfers. The red curves in Figure 2 illustrate the set of all transfers that satisfy the government’s budget constraint. The key point is that this set is a “curve” for $t_1 > Z$. Starting from point A, where transfers are zero, along the dashed red curve, as $t_1$ exceeds $Z$, $t_2$ must become negative to satisfy the budget constraint. However, with negative date 2 transfers, a fraction of borrowers will default, and increasingly so as $t_2$ becomes more negative; this induces curvature in the government’s budget set. We also graph the isoquants for the liquidity-constrained high-default-cost household. Taking only this household into account, we see that at the optimum point B, the planner sets $t_1 > 0$ and $t_2 < 0$. Accounting for the utility of the household that defaults increases $t_1$ further since this household places weight only on the date 1 transfer. As $Z$ rises, the dashed red curve shifts out to the solid red curve, and at the tangency point C, the transfer $t_1$ becomes larger, while the required repayment $t_2$ falls. Thus the contract calls for payment reduction and payment deferral, with more reduction available as $Z$ rises. (Later, we allow for the default cost $\phi$ to be unobserved to the policymaker and lender, so that adverse selection is an issue.)

### 3.3 Principal reduction with default risk

Above we considered the case where $t_1 > 0$ and $t_2 < 0$. In the case of principal reduction, both $t_1$ and $t_2$ are positive. In particular, since $t_2 > 0$, the planner transfers resources to the household and the budget constraint is,

$$Z - t_2 - t_1 = 0 \quad (24)$$

Suppose we solve the planning problem subject to the above budget constraint and restrict attention to solutions where $t_1$ and $t_2$ are non-negative. Figure 3 illustrates the solution. The dashed area illustrates the set of all points such that $t_1 + t_2 = Z, t_1 > 0, t_2 > 0$. It is clear that the solution is a corner: set $t_1 = Z$ and $t_2 = 0$ (point A in the figure). This implies that principal reduction (in which $t_2 > 0$) is not optimal, since the solution goes to the corner where the transfers are front-loaded, that is, for payment reduction focused in period 1. This occurs despite the fact that our problem allows for strategic default with default costs, and that borrowers default less if $t_2 > 0$. For high enough $Z$, the transfer to date 1 is sufficient to ensure full consumption smoothing, and hence there is no need for further transfers.

In this setting, principal reduction is never optimal, even though default is costly and is accounted for
by the planner, because the alternative of directly transferring the same resources to households in the first period raises utility more. It is optimal for the planner to use this strategy until the liquidity constraint no longer binds, and complete consumption smoothing is achieved. Until that occurs, principal reduction is suboptimal compared to payment deferral or reduction, and thereafter no policy intervention is needed to address liquidity constraints.

Finally, while we have solved a planning problem to demonstrate that principal reduction is not optimal, we also find the same result in a problem where a government offers transfers to households and then lets households trade with private lenders to achieve their optimal solution.\footnote{Consider private lender transactions where we assume that private agents face the same discount rate as the government, even in the crisis. If the government can access credit markets at a lower rate than private agents, our results are strengthened.}
that at least break even for the lenders, i.e.,

\[-\tau_2(1 - F(-\tau_2)) - \tau_1 = 0.\]

Suppose that that the government offers a pure date 2 transfer of $Z$. In Figure 3, we represent this by moving from the zero transfer allocation to point $B$. The red curve in Figure 3 represents the set of trades, $(\tau_1, \tau_2)$, that a private sector lender will make that allows the lender to break even. This allows agents to borrow against the future transfer $Z$ in order to smooth consumption, solving the liquidity constraint problem at date 1. Again, the critical thing to note is that the borrowing constraint becomes a curve. Starting from point $B$, the household will trade to point $C$, which achieves less utility than point $A$. That is, the household will choose to borrow the $Z$ back to increase date 1 consumption. However, since some borrowers default, the interest rate on the private loan will exceed one so that the government would do better by offering the transfer of $Z$ at date 1, i.e., a payment reduction rather than the principal reduction, to reach point $A$. This is a general point to which we return later. Even if principal reduction is sufficiently generous to overcome individual borrowing constraints, direct payouts to borrowers are more efficient as the government avoids default costs.\(^\text{12}\)

The key insight underlying these results is the constraint affecting date 1 consumption. Even if credit markets exist to transfer date 2 resources into date 1 consumption, default risk makes this approach more expensive than a direct date 1 transfer to households. Hence, even with default risk, we again find that transfers in the initial crisis period at least weakly dominate policies that transfer resources later.

4 Default Timing and Loan Modifications

We now examine the case where a borrower may choose to default at date 1 given information $E\phi \equiv E_{t=1}[\phi]$, but may also default at date 2. In this case, the borrower has a timing decision as well as a default decision.

With no default in the initial period, utility at date 1 is,

\[
\left(\frac{\alpha \bar{y}}{\tau}\right)^\alpha (y_1 + t_1 - \alpha \bar{y})^{1-\alpha}
\]

\(^\text{12}\)In general, principal reduction to reduce the underwater share of mortgages takes borrowers at most to LTV (loan to value) of 100, which does not generally create borrowing capacity. Even if it did, as we allow above, our analysis shows that direct transfers at date 1 remain more efficient.
If the household defaults at date 1, she can reoptimize her consumption plan to rebalance housing and non-housing consumption, giving utility of,

\[ y_1 \left( \left( \frac{\alpha}{\tau} \right)^\alpha (1-\alpha)^{1-\alpha} \right) = y_1 \psi, \quad \text{where} \quad \psi \equiv \left( \frac{\alpha}{\tau} \right)^\alpha (1-\alpha)^{1-\alpha}. \]  

(26)

However, if the household defaults at date 1, he loses any value in his home as well as the option to default at date 2. Under default at date 1, date 2 wealth becomes \( \bar{y} - \theta \), giving date 2 utility of

\[ (\bar{y} - \theta)\psi. \]

With no default at date 1, utility at date 2 is

\[ [\bar{y} + E \max (P_2 + t_2 - D, -\theta)] \psi. \]  

(27)
Hence, comparing values with and without date 1 default, default at date 1 occurs if

\[ y_1 \psi + (\bar{y} - \theta) \psi > \left( \frac{\alpha \bar{y}}{\gamma} \right)^{1-\alpha} (y_1 + t_1 - \alpha \bar{y})^{1-\alpha} + E[\bar{y} + \max(P_2 + t_2 - D, -\theta)] \psi. \]  

(28)

Rewriting, we obtain the condition under which default occurs at date 1 as

\[ y_1 - \bar{y} \left( \frac{y_1 + t_1 - \alpha}{y} \right)^{1-\alpha} > E[\max(t_2 + \phi, 0)]. \]  

(29)

Figure 4 graphs the left and right hand side of (29) as a function of \( E\phi \), which measures the degree to which a homeowner has equity \( (P_2 - D) \) (plus the default cost), or the inverse of "underwaterness". The blue line graphs the value of the option to keep making mortgage payments and delaying default, on the right-hand side of equation (29). This value is uniformly positive, although low for low values of \( E\phi \). The red line is the benefit of defaulting, on the left-hand side of equation (29). This value is independent of \( E\phi \). For low values of \( E\phi \), the household chooses to default at date 1. A simple way to think about the default decision at date 1 is as follows: underwater homeowners have a call option on the home, which is extinguished by default. Thus the choice to make the mortgage payment at date 1 is a question of whether the cost of making this payment covers the value of the call option. When liquidity constraints are tight, the cost of making the payment is highest; this determines the height of the horizontal red line in Figure 4. When the borrower is underwater, the value of the call option is lowest, as shown in the blue line, which rises as the household’s equity in the home rises. The intersection of the red and blue lines, at point A, determines the value of \( E\phi \), or the degree of being underwater, that triggers default. This characterization is also consistent with the "double-trigger" model of default, as in Foote, Gerardi, and Willen (2008), for example: underwater, liquidity-constrained homeowners are the most likely to default.

The figure also illustrates how the default decision depends on \( t_1 \) and \( t_2 \). Increasing \( t_1 \) shifts down the benefit to defaulting (in red) at all values of \( E\phi \) to the dashed red line. Hence the trigger value falls from point A to point C; the household will be more deeply underwater before defaulting. Increasing \( t_2 \) increases the cost of defaulting, shifting up the blue curve to the dashed blue curve, and the trigger value falls from point A to point B. Note that this latter effect is strongest at higher values of \( E\phi \), on the right-hand side of Figure 4. However, this is the region for which default is dominated; the default option is out of the money. Hence, positive date 2 transfers move equity values most when households are least likely to default. This

\[ ^{13} \text{This is the same intuition as in the Leland (1994) model of dynamic corporate capital structure.} \]
reinforces our earlier finding that date 1 transfers are more effective than transfers made later. This point can be seen clearly analytically. The derivative of the left-hand side of equation (29) with respect to \( t_1 \) is,

\[
\frac{d}{dt_1} \left( y_1 + t_1 y_1 \right) < 1
\]

The derivative of the right-hand side of equation (29) with respect to \( t_2 \) is,

\[
\frac{\partial}{\partial t_2} \int_{-t_2}^{\infty} t_2 dF(\phi) = F(-t_2) < 1.
\]

Thus a dollar increase in \( t_1 \) always decreases the benefit of defaulting at date 1 more than a dollar increase in \( t_2 \). The difference in these effects increases as \( E\phi \) falls, that is, as the mortgage is more underwater. Hence, the more underwater is the loan, the more effective is initial payment reduction at avoiding default, relative
to an equivalent transfer received at date 2. This reinforces our earlier finding that date 1 transfers are more effective than flat or back-loaded transfers. Initially, this was clear with a date 1 liquidity constraint, but the same result obtains with date 2 default and now with the possibility of date 1 default and default timing on strategic default.

This approach also illustrates the role of uncertainty, which raises the option value of waiting, or in terms of Figure 4, shifts up the blue curve. The piecewise linear black line gives the payoff value under certainty (when $\phi$ is known); greater uncertainty shifts the blue curve up relative to the black piecewise line. Higher home price uncertainty is therefore associated with fewer defaults at date 1, as homeowners have a greater option value of waiting for home prices to rise. This illustrates the subtlety of arguments about the effect of uncertainty on the economy. Putting a floor under home prices (reducing the mass in the left tail) would reduce defaults, but reducing uncertainty, or trading off a floor with a commensurate ceiling on home prices, could increase defaults.\textsuperscript{14}

Finally, we note that a borrower who does not experience an income shock, $y_1 = \bar{y}$, never defaults at date 1. The left hand side of (29) is zero in this case, because there is no benefit to reoptimizing date 1 consumption. Moreover, the right hand side is strictly positive. Even in the case where $\phi$ is expected to be negative, there is a positive value to waiting and exercising the option to strategically default at date 2, so that it is never optimal to default at date 1. This cleanly illustrates the intuition for strategic delay by unconstrained households.

We conclude from this analysis that payment reductions at date 1 are more effective than flat or back-loaded transfers in preventing default at date 1. Principal reductions at date 2 are most effective in preventing strategic default at date 2. This finding reinforces our earlier results for liquidity constrained households. There, the binding liquidity constraint made it clear that for macroeconomic consumption purposes, date 1 transfers are the most effective use of government budget resources. Allowing for future default modified this finding: date 1 payments coupled with repayment at date 2 can induce default at date 2. Hence, payments

\textsuperscript{14}The latter effect is likely to dominate in fact. Since the household defaults when the home price outlook is particularly bleak, the details of the left tail distribution do not matter for behavior. That is, the details of bad outcomes do not matter to the household since it defaults in those states. However, the borrower does not default when home prices are expected to improve, so the upper tail is relevant for forward-looking decision-making. This is a generalization of Bernanke’s (1983) "bad news principle" in the two-sided setting of Abel, Dixit, Eberly, and Pindyck (1996). Here, we have a "good news principle" for borrowers because they have a default, or a put, option.
should be flatter but still front-loaded. A flat or back-loaded transfer schedule is always dominated by date 1 payments until the liquidity constraint is fully relaxed.\textsuperscript{15} Hence, with default and an option value of delay, we still obtain that transfers in the initial crisis period at least weakly dominate policies that transfer resources later.

5 Lender Renegotiation to Reduce Strategic Default

We have shown that government resources aimed at reducing default are better spent on payment reduction than principal reduction, largely because underwater mortgages are substantially out of the money so that decreasing debt levels has a small effect on default incentives. However, this same logic increases lender incentives to renegotiate down debt, as we now show. Moreover, these incentives are highest when the borrower is underwater on his mortgage.

Consider a borrower at date 2 whose home price exceeds the mortgage amount less the deadweight cost of default. This borrower is expected to repay, and hence the lender receives the loan amount, $D$. On the other hand, consider a borrower with $\phi < 0$ at date 2, who as a result will be expected to default on his debt. In that case, the lender receives the home which is worth $P_2$.\textsuperscript{16} Denote $V_2(P_2, D)$ as the value of the mortgage loan to a lender conditional on a given price $P_2$ and debt level $D$. Then,

\begin{align*}
  &\text{If } P_2 \geq D - \theta \Rightarrow V_2(P_2, D) = D \\
  &\text{If } P_2 < D - \theta \Rightarrow V_2(P_2, D) = P_2.
\end{align*}

Figure 5 graphs $V_2(\cdot)$ as a function of $P_2$ for two levels of debt, $D$ and $D'$ (where $D' < D$). The comparison illustrates that when $P_2 < D - \theta$ (that is, $\phi < 0$), the lender can increase the value of its loan by reducing $D$ to $D'$. This occurs because $\theta$ is a deadweight cost of default. If the borrower defaults on his loan,\textsuperscript{15,16}

\textsuperscript{15}Our analysis assumes that the income shock is temporary, which is the interesting case for policy analysis to avoid default. If a shock is permanent but not common to all households, then default may be optimal as reallocation is necessary. In that case, optimal policy may still favor delay (if there is still price or other uncertainty to be resolved or the price elasticity of foreclosures declines over time). Government policy may also favor less-disruptive forms of default, such as short sales or rental-in-place arrangements, that can reduce the deadweight cost of default. Policy may also encourage lender renegotiation by giving more bargaining power to borrowers in these instances, through legal procedures such as bankruptcy and cramdown.

\textsuperscript{16}We assume that the lender has the bargaining power in renegotiation. In intermediate cases of shared bargaining power, the results would depend on the allocation of bargaining power, but the general findings would still hold.
the lender only collects $P_2$. However, the borrower’s value of keeping the loan and not defaulting is $P_2 + \theta$. Thus, the lender can offer to write down the principal to $D' < D$ and still increase the value of its loan.\(^\text{17}\) In this case principal reduction leads to better date 2 outcomes. Note, however, that no government resources are required to implement the principal reduction, as it is privately optimal since the lender benefits from avoiding default. As we point out later, though, the government may play an important role in encouraging and coordinating the renegotiation (for example, by standardizing the structure of modifications).

\(^\text{17}\)In this analysis we are ignoring the fact that \textit{ex post} loan forgiveness implies that lenders will thereafter expect loan forgiveness and price it into subsequent contracts, making credit more expensive. At this point, however, our intention is to examine under what circumstances even \textit{ex post} loan forgiveness may make sense. We return to this topic later when we discuss \textit{ex ante} security design.
to do so at date 2. To examine this question, suppose $P_t$ is the value of the home, which is a random variable that evolves over the interval $t \in [1, 2]$. Denote $V_t(P_t, D) \equiv E_t[V_2(P_2, D)|P_t]$ as the expected value of the mortgage to the lender. From Figure 5, we see that reducing $D$ has two effects on value: it reduces the payment to the lender in the non-default states of the world, with the reduction in value indicated by the red area. That is, if the borrower pays in full, then reducing principal just reduces the payment to the lender. However, principal reduction also increase payments in some of the default states, by preventing default, with the increase indicated by the green area. It is easy to see that if $P_t$ is very high, so that the probability of $P_2$ falling in the red area is large, then $\frac{\partial V_t(P_t, D)}{\partial D} > 0$, and principal reductions reduce lender value. On the other hand, if $P_t < D - \theta$ and the probability mass of $P_2$ is highest in the green area compared to the red area, then principal reductions increase value. This is likely to be true if $P_t < D - \theta$ and $t$ is either near $t = 2$ or the volatility of home prices is low. In practice, this suggests the principal reduction is most attractive to lenders when loans are deeply underwater (by an amount in excess of borrower’s default costs), when borrowers are near default, and when home price volatility is low.

As in the previous section, these effects can be understood in terms of an American option, though here from the lender’s perspective. By writing down principal, the lender extinguishes the option to write down later.\footnote{In principle, loans could be written down more than once, but if it is costly to do so, then the value of delay still obtains.} It is rarely optimal to exercise the option early, and the value of the option is greater with higher uncertainty. Hence, lenders would have an incentive to wait to write down principal, but would wait longer the greater was the volatility around the value of default (including net equity and home prices). Hence, even if $\frac{\partial V_t(P_t, D)}{\partial D} < 0$ so that early principal reductions increase value, it is easy to verify that a lender would always do better by waiting until $t = 2$ and deciding on a principal reduction. This is because by waiting until $t = 2$ the lender can make the reduction contingent on whether or not $P_2$ is below or above $D - \theta$. This effect is reinforced by any government transfer that decreases the incentive for default in Figure 4, and hence reduces the incentive for a private lender renegotiation. Hence, a government writedown makes a private writedown less likely.\footnote{Moreover, we have not modeled the payment stream, but in practice, for loans not in distress, the lender continues to collect the interest and principal on the higher loan balance.}

In practice, however, there may be costs of waiting. It may take time to process the contractual requirements of reducing loan principal. Prices may move discretely and the borrower defaults before the lender is
able to implement the reduction. Such considerations may lead the lender to reduce principal preemptively, though the value of delay will always be balanced against those considerations.

Finally, in practice lenders were not active in doing principal reductions, though later principal reductions were used more as part of lender loan modifications, especially by specialty servicers. Other considerations included reputational effects and incentives affecting a lender’s whole portfolio of loans, rather than just individual borrowers. For loans not held on balance sheet by lenders, servicer incentives and capacity may also have reinforced delay and timing discreteness. Our theoretical findings are consistent with the empirical work of Adelino, Gerardi, and Willen (2013), who document the reluctance of servicers to renegotiate mortgages and emphasize the presence of uncertainty arising from redefault risk and self-curing of mortgage delinquencies. Other authors address administrative and structural frictions to loan renegotiation and recommend legal and policy changes to reduce them, for example, Mayer, Morrison and Piskorski (2009) and Geanakoplos and Koniak (2011). The efficacy of these proposals is outside our present scope, though the challenges of servicers and the administrative structure of mortgages also point to the desirability of \textit{ex ante} reforms, as opposed to \textit{ex post} renegotiations.

The last two sections demonstrate that delay can be desirable to both borrowers and lenders, who see default as extinguishing a valuable option to wait and possibly avoid costly foreclosure. Government policy to “speed things up” can be problematic in this setting, since it may result in more foreclosures rather than fewer. Nonetheless, the government may still intervene if it values the externalities associated with foreclosure more than private agents do, and hence moves more quickly to address inefficient servicer delays, information problems, and capacity constraints.

5.1 Refinancing and principal reduction

These effects are avoided by including the modification option in the contract \textit{ex ante}. In fact, lenders and borrowers often achieved the equivalent of principal reduction through mortgage refinancings, some of which were contracted \textit{ex ante} (through the prepayment option) and some of which were allowed \textit{ex post} (for underwater loans which could not be prepaid in practice from a new mortgage on the existing collateral) through government support.

To see the formal equivalence between payment streams under principal reductions and refinancing, note
that both forms of restructuring loans reduce the stream of payments on the mortgage over time, rather than front-loading the benefits, as suggested by our analysis of liquidity constraints. However, for a given fixed rate, fixed term loan, any new stream of payments that can be achieved with a reduction in face value can also be achieved by a reduction in the contract interest rate. This parallel between principal reduction and refinancing is largely unnoticed because refinancing does not change the face value (principal) of the loan, while principal writedowns explicitly reduce the face value. This is misleading, however, because face value is a poor measure of the value of a loan. On a market value basis, refinancing to a lower interest rate reduces the value of the loan. Mortgage lenders and investors see the effect in market valuations, and borrowers see the effect in their payments. A reduction in the payment stream achieved through a reduction in face value can always be replicated by a change in the contract interest rate. For example, and to get a sense of magnitudes, a refinancing of a 30 year $200,000 mortgage from 6 percent to a 4 percent interest rate, reduces monthly payments from $1200 to $950 (20 percent), and the present value of the stream of payments from $250,000 to its face value of $200,000. The identical payment stream would result from a reduction in face value, or a principal write down, from $200,000 to $160,000 (or 20 percent).

Interestingly, refinancings generally occurred during the financial crisis in two ways. Either borrowers had positive equity and could refinance in a competitive market; these are unconstrained borrowers in our setting. This would have been possible regardless of the housing collapse. Alternatively, underwater borrowers from the GSEs, Fannie Mae and Freddie Mac, who were current on their loans, could refinance through the HARP program, and a similar option was made available to some non-GSE borrowers under the National Mortgage Servicing Settlement. These borrowers were also arguably unconstrained, in that they were making their payments on time and were not in payment distress. Such circumstances fit the model’s recommendation for implementing principal reduction for unconstrained borrowers to avoid strategic default. (Because the

20 While the present value of the payment streams can be equated, the time path will differ. In particular, a written-down loan will have a lower initial pay-off value, while a refinanced loan will have a lower pay-off value than the original loan, but will amortize the lower pay-off value over time. The distinction does not affect the incentive for strategic default (since the pay-off value is not paid in case of default). However, it can matter for the incentive or ability to prepay the loan. Hence, writedowns may tend to increase turnover more than refinances.

21 Refinancings often further reduce payments by extending the term of the loan, but that would confound the effects of the interest rate reduction and the term extension in this example, without changing the essential point.

22 The terms of the National Mortgage Servicing Settlement are described here by the settlement monitor: https://www.jasmithmonitoring.com/omso/reports/final-crediting-report/
program was made available by the GSEs to borrowers directly, lenders/investors did not have the option to delay.) Through HARP, borrowers received $t_1 > 0$ and $t_2 > 0$, financed by a reduction in the mortgage value held by lenders/investors. We return to this point in the policy section of the paper.

5.2 Debt overhang, writedowns and loan renegotiation

The debt overhang from underwater mortgages is an additional macroeconomic consideration, as continuing to make mortgage payments prevents households from rebalancing their spending toward other forms of consumption, as emphasized by Dynan (2012). Hence, in addition to reducing default, principal writedowns may also ease a debt overhang problem by easing borrower’s date 1 credit constraint. If the government would prefer to increase date 1 consumption, easing the credit constraint could be desirable. Does this change the calculus of government interventions to ease the liquidity constraint; that is, does debt overhang suggest that principal reduction is valuable over and above elimination of deadweight loss?

The answer is no. Suppose at date 1 the government offers a loan modification of $t_2 > 0, t_1 = 0$, to reduce principal by $t_2$. (We structure the modification in this way to be clear that any increase in date 1 resources comes from easing the debt overhang and not from a direct government transfer at date 1.) Then at date 2 lenders offer a principal writedown to those borrowers that are \textit{ex post} revealed to have $\phi < 0$ (negative equity) in order to prevent strategic default. This structure allows the government to reduce the debt overhang by reducing loan balances, and the private principal writedown layers on additional debt reduction for underwater borrowers.\textsuperscript{23} As we allowed previously, now assume that the debt writedowns are sufficiently generous to overcome the liquidity constraint at date 1. Hence, now let the household borrow in the private market to achieve consumption smoothing and increase date 1 consumption, taking advantage of the elimination of the debt overhang and expected date 2 elimination of negative equity.\textsuperscript{24}

\textsuperscript{23}We show later that competition among lenders could result in an unraveling of modifications due to adverse selection. The same result does not apply here, since lenders will not compete to write down the debt of underwater mortgages. The only lender willing to write down debt is the lender subject to the default risk. Hence, there is no unraveling of principal reduction offers in the way that there could be with payment modifications, where a restructured loan could be valuable. In practice, some specialized servicers purchased distressed loans at steep discounts and modified them, including principal reduction. However, this was only after the loan had already been written down by the original lender at the sale.

\textsuperscript{24}We have assumed that the principal write down is sufficiently large to eliminate the credit constraint. In practice, borrowers would typically need positive equity in their homes or other wealth to access credit markets. Here we assume that eliminating
constraint for the private lenders requires that the repayment, \( \tau_2 \), for the fraction of borrowers who repay, 
\((1 - \hat{F}(-\tau_2))\), be sufficient to cover the date 1 loans, \( \tau_1 \):

\[-\tau_2(1 - \hat{F}(-\tau_2)) - \tau_1 = 0,\]

where \( \hat{F}(\phi) \) is the distribution of net equity \( \phi \), anticipating renegotiation between the borrower and lender and resulting debt write downs. At \( \tau_2 = 0 \), we have \( \hat{F}(0) = 0 \) so that the household is able to take the first dollar of loan at a zero interest rate. However, as the required repayment \( \tau_2 \) rises, some households will default and the interest rate on the marginal loan rises above zero, in order to compensate the lender for these expected defaults.\(^{25}\) Thus the government would be better off using these resources to set \( t_1 > 0 \) and \( t_2 = 0 \), which is a payment reduction to directly ease liquidity constraints. Government resources to reduce principal are better spent in engaging lenders to renegotiate mortgage loans, rather than writing them down directly.\(^{26}\)

Before turning to equilibrium, we can briefly summarize our findings so far on modifications, which imply two broad results. First, with liquidity constraints, transfers to households at date 1 weakly dominate transfers at later dates and hence are a more effective use of government resources. Date 1 transfers could include temporary payment reductions, such as interest rate reductions, payment deferral, term extensions. This result is robust to including default, various forms of deadweight costs of default, debt overhang, and the easing of credit constraints through principal reduction. Any policy that transfers resources later can be replicated by an initial transfer of resources, but the converse is not true. Second, principal reductions offered by the private sector can reduce any deadweight costs due to strategic default. This conclusion is independent of whether or not liquidity constraints are present. With the potential for delay, these defaults, and hence the debt writedowns would occur at date 2.

\(^{25}\)If \( \tau_2 \) is also forgiven at date 2, similar to the home mortgage debt writedowns, then the borrower can raise no money on this private loan and \( \tau_1 = 0 \). This reinforces the finding that principal reduction does not ease liquidity constraints. Effectively writing down household debt with the expectation that households will be able to borrow more is difficult to justify conceptually. This example also illustrates the negative ex-ante consequences of forgiving loans ex-post.

\(^{26}\)Note that we have not assumed that the government has a lower cost of capital than private agents. This result relies only on the fact that by transferring resources at date 1, the government directly relaxes the liquidity constraint. Whereas, date 2 resources require the agent to borrow and transfer them to date 1. With any default risk, the price to agents of doing so will exceed the cost of the direct transfer.
6 Housing Market Equilibrium and the Effect of Foreclosures

So far we have allowed uncertainty in home prices but not endogeneity. An additional reason to modify loans and reduce default might be to intervene in the dynamic equilibrium from default to home prices and back to default, as documented empirically by Harding, Rosenblatt, and Yao (2008), Campbell, Giglio, and Pathak (2011), Mian, Sufi, and Trebbi (2011), and Anenberg and Kung (2014). Hence, we are interested in understanding how default/foreclosures at date 1 and/or date 2 affect housing prices. This section sketches a minimal general equilibrium of the housing market to clarify how such considerations may alter our conclusions regarding modifications.

Denote $p_t$ as the price per unit housing. Earlier, we described a household purchasing $c^h_t$ units of housing services at price $P_t$, so that the price per unit of housing was $p_t = \frac{P_t}{c^h_t}$. Equivalently, $p_t$ is the price of a normalized quantity of a house of size “one.” Then,

$$p_0 = E[r_1 + r_2 + p_2]$$

Here $r_1$ and $r_2$ are the date 1 and date 2 user cost of housing, respectively.

Next we close the model to specify a housing market equilibrium that determines $p_0$. We follow our initial framework and assume that at a planning stage, households anticipate income of $\bar{y}$ at both dates, and choose housing consumption,

$$c^h_t = \alpha \frac{\bar{y}}{r_t}$$

At date 1, income of the households falls to $y_1$. For now, we assume that exogenously a fraction $m_{1,L}$ of the households default on their mortgages and enter the rental market, where $L$ denotes the liquidity constrained households. We can think of the households subject to the income shock and default as the liquidity constrained households we modeled earlier.

We depart from our previous assumption and allow for a degree of friction in the rental markets so that owning a home is more efficient than renting a home. To purchase one unit of housing services costs $r_t$ in debt service, while to generate the same housing services costs via the rental market, it costs $f r_t$, where $f \geq 1$, and $f$ parameterizes the rental friction, with $f = 1$ being the case we have analyzed in previous

\[27\] This may capture moral hazard or other information problems associated with the rental market.
section. Thus, the date 1 demand for housing via the rental market from the foreclosed homeowners is,
\[ c^h_1 = \alpha \frac{y_1}{fr_1} \]
We assume that foreclosure keeps the household out of the ownership market for one period. At date 2, the household purchases a home again so that,
\[ c^h_2 = \alpha \frac{\bar{y}}{r_2} \]
Suppose that at date 1, across the economy there are \( m_{1,L} \) agents renting, and \( 1 - m_{1,L} \) agents owning. Then total demand for housing at date 1 from these agents is,
\[ m_{1,L} \frac{y_1}{fr_1} + (1 - m_{1,L}) \frac{\bar{y}}{r_1} = \alpha \frac{\bar{y}}{r_1} - m_{1,L} \frac{\alpha}{r_1} \left( \frac{\bar{y} - y_1}{f} \right) . \]
Foreclosures, i.e., an increase in \( m_{1,L} \), decreases the net demand for housing at date 1. At date 2, since the date 1 foreclosed homeowners own homes again, the total demand for housing is \( \alpha \frac{\bar{y}}{r_2} \) and invariant to \( m_{1,L} \).

We assume that there are other unmodeled agents in the economy who also consume housing services. These may include new home buyers, home builders, speculators, etc. We denote the demand from these agents as \( H^D(r_1) \). Our modeling only takes a stand on the functional forms for the households who are subject to foreclosures and who will be affected by modifications. The market clearing condition at date 1 is,
\[ H^D(r_1) + \alpha \frac{\bar{y}}{r_1} - m_{1,L} \frac{\alpha}{r_1} \left( \frac{\bar{y} - y_1}{f} \right) = H \]
where the supply of housing is fixed at \( H \). The housing dividend, \( r_1 \), is increasing in the income of homeowners and renters, decreasing in \( H \) and decreasing in \( m_{1,L} \).

### 6.1 Effect of foreclosures

Let us now consider a rising foreclosure scenario where \( m_{1,L} \) increases. If \( dm_{1,L} \) agents switch from owning to renting, then effective consumption of housing services in the economy falls and housing prices will fall. Define,
\[ \eta_1 = \frac{1}{r_1} \frac{dr_1}{dH} < 0 \]
as the percentage change in the housing dividend for a unit increase in housing supply. \( \eta_1 \) is the reciprocal of the semi-price elasticity of demand. This derivative should be interpreted as the percentage reduction in
price caused by the sale of an additional unit of housing (“price pressure”). Then,

\[ \Delta p^{1,L} = \frac{dp}{dm_{1,L}} = \eta_1 \alpha \left( \bar{y} - \frac{y_1}{f} \right) < 0 \]

We can also consider a foreclosure scenario for strategic defaulters. These households differ from the constrained households because their income at date 1 is \( \bar{y} \) but they nonetheless default on mortgages whose face value is higher than the home price, including default costs. Going back through the derivation of housing demand for strategic defaulting homeowners becoming renters, we find that the net demand for housing as a function of the number of strategically defaulting renters, \( m_{1,S} \), is,

\[ \frac{\alpha \bar{y}}{r_1} - \frac{\alpha}{r_1} \left( \bar{y} - \frac{\bar{y}}{f} \right). \]

So that \( \frac{dp}{dm_{1,S}} \) is,

\[ \Delta p^{1,S} = \eta_1 \alpha \left( \bar{y} - \frac{\bar{y}}{f} \right) > \Delta p^{1,L} \]

Thus foreclosures reduce prices more in the case of liquidity constraints than in the case of strategic defaults. This occurs because the liquidity constrained defaulters experience a larger drop in the net demand for housing services when going from owning to renting. Note also that if \( f = 1 \) so that the rental market is frictionless, then \( \Delta p^{1,S} = 0 \), and \( \Delta p^{1,L} < 0 \).

We can repeat the same exercise at date 2. At date 2 all default is strategic. Thus the effect of foreclosures at date 2 is

\[ \Delta p^{2,S} = \eta_2 \alpha \left( \bar{y} - \frac{\bar{y}}{f} \right) \]

It is likely that the housing market is more stressed at date 1 in a recession than at date 2, so the home sales have bigger price impact at date 1 than at date 2 and \( \eta_1 > \eta_2 \). In this case, it follows that we can order the effect of foreclosures on home prices as

\[ |\Delta p^{1,L}| > |\Delta p^{1,S}| > |\Delta p^{2,S}|. \]

That is, the effect on home prices is largest from a liquidity-constraint induced default, which occur during the crisis period (date 1) in our model. Strategic defaults also put downward pressure on home prices, but less because these homeowners do not bring an income shock and payment distress into the rental market. This effect eases once the economy moves out of the crisis period.
6.2 Mortgage modifications and home prices

We now revisit how the proposed mortgage modifications, parameterized by \( t_1 \) and \( t_2 \), affect home prices. We showed earlier that payment reductions, i.e., \( t_1 > 0 \), are most efficient in reducing default at date 1 when households are liquidity constrained. Home prices are also most sensitive to defaults of liquidity constrained agents. Putting these findings together, we conclude that if a planner’s objective includes home price stabilization (or if home prices feed back to consumption and utility), then payment reductions for liquidity constrained agents are a more effective tool than principal reductions. Thus payment reductions should be optimally targeted at liquidity constrained agents. This finding reinforces our earlier conclusions on the benefits of payment reductions for liquidity constrained agents.

Principal reductions, i.e., \( t_2 > 0 \), are less effective in reducing default at date 1, but do help in reducing strategic default at date 2. Reducing strategic default at date 2 also stabilizes home prices, albeit less strongly than at date 1.

Hence, endogenous home prices reinforce our general finding that transfers during the crisis period at least weakly dominate policies that transfer resources to households at later dates. Endogenous home prices also suggest that there is value to associating these transfers with housing. That is, our earlier results, which just focused on consumption smoothing, could have been accomplished with any type of transfer that supports consumption spending. With endogenous home prices, there is additional value to supporting spending on housing, specifically, to prevent foreclosures and the negative effect on home prices.

7 Implications for Ex Post Policy Choices and Ex Ante Security Design

These results can be used to examine both ex post policies, as we have described above, and ex ante policies. The former apply once a housing crisis is underway and policy-makers must decide if and how to react. In this setting, we consider only the effects of the policies at the time of implementation and not any subsequent effect on the cost of credit. If loans are written down ex post, the cost of credit may rise as lenders price in the probability of future writedowns. In this analysis our intention is to examine the impact of the writedowns themselves. If these problems can be anticipated, however, policies can be put in place ex ante
to ease the policy choices faced in the midst of a crisis and avoid these *ex post* implications.

### 7.1 Adverse selection and *ex post* unraveling

Our framework suggests that policies to focus reduced payments at date 1 improve macroeconomic outcomes, including consumption, mortgage default, and home prices. In the model with default costs, lenders should be willing to offer payment reductions at date 1, in order to improve loan performance, while liquidity constrained borrowers would be willing to pay more at date 2 in order to have this concession at date 1. In other words, such a contract should be privately optimal. These modifications could have taken the form of payment reductions with payment deferral and/or term extensions. In fact, loan modifications or renegotiations of this type, or indeed any type, were rare, especially early in crisis (Agarwal, *et al*, 2011). If households were liquidity constrained and their mortgage in distress, why were these arrangements not more common? One possible explanation is adverse selection.

Returning to our model with unknown default costs from Section 3.2, now suppose now that $\phi$ is private information of the borrowers. In addition, suppose that $1 - \lambda$ fraction of the households are liquidity constrained as described, but $\lambda$ fraction are unconstrained. In particular, for these unconstrained households $y_1 = \bar{y}$, so that they do not have to cut back on consumption at date 1.

Let us focus on a modification program with $t_1 > 0$ and $t_2 < 0$ where, $v'(y_1 - \alpha \bar{y} + t_1) > \psi$. That is, the terms of this program are such that all liquidity constrained households find it beneficial to take the program. On the other hand, among unconstrained households, only those with low default costs, $\phi < -t_2$, will take the loan. For this household, the modification, or consumption loan, is a free transfer of $t_1$ since the household does not intend to repay the loan. For a high default cost household that is unconstrained, the loan is not useful; it does not increase utility because consumption is already smooth across periods and the terms of trade in the loan imply an interest rate above one. Then, within the population of households who accept modifications, the fraction of defaulters $F^A$ is,

$$F^A(-t_2) = \frac{\lambda F(-t_2) + (1 - \lambda) F(-t_2)}{\lambda F(-t_2) + (1 - \lambda)} > F(-t_2)$$

The breakeven condition under which a lender would offer the loan requires that,

$$Z - t_2(1 - F^A(-t_2)) - t_1 = 0.$$
Hence, the larger the fraction of defaulters, \( F^A \), the smaller the initial amount of liquidity to support consumption, \( t_1 \), for given \( t_2 \). As the share of unconstrained households, \( \lambda \), rises, \( F^A(-t_2) \) goes to one, and the effective interest required for a lender not to lose money goes to infinity. In other words, the unconstrained strategic defaulters drive up the cost of the modification for liquidity constrained borrowers. At higher interest rates, the liquidity constrained borrowers also self-select: only low default cost households take the loan and hence the fraction of defaulters in the population goes towards one. For sufficiently high \( \lambda \), the modification market breaks down for standard lemons market reasons: the only contract offered is \( t_1 = t_2 = 0 \).

We can again write a planning problem to derive the optimal \((t_1, t_2)\). The solution calls for \( t_1 > 0 \) and \( t_2 < 0 \), following the same logic as the previous case. As \( \lambda \) rises there are more strategic defaulters in the pool, and the solution requires a smaller initial transfer \( t_1 \).

Now suppose that modifications are instead done by the private sector rather than a government. Consider two lenders engaged in Bertrand competition. Fix a modification contract \((t_1, t_2)\) such that the lenders each break even. Now suppose that one of the lenders offers a contract \( \hat{t}_1 = t_1 - \epsilon_1 \) with \( \hat{t}_2 = \frac{t_2}{t_1} \hat{t}_1 + \epsilon_2 \), for positive and small \( \epsilon_1, \epsilon_2 \). The second contract involves a smaller date 1 loan, but also a smaller interest rate on the loan. The contract is not attractive to the unconstrained borrowers because they will not repay, and hence care only about the size of the modification and not the effective interest rate. But we can always choose \( \epsilon_1 \) and \( \epsilon_2 \) such that the liquidity constrained borrowers prefer the second contract over the first contract. That is, the interest rate savings, \( \epsilon_2 \), can be chosen to be large enough to compensate for the reduction in loan size, \( \epsilon_1 \), to make this contract preferred by liquidity constrained borrowers. In this case, the second contract is a profitable deviation by a lender. But as a result the initial lender loses money, as this lender is left with a population of unconstrained strategic defaulters; he will therefore lose \( t_1 \). The first lender will then have to match the second lender and reduce \( t_1 \), but this offer will also be undercut. Equilibrium can unravel in the sense of Rothschild-Stiglitz (1976).

This logic provides two insights. First, it offers one reason why modifications were not offered more widely. Competition and the fear of receiving an adverse pool of borrowers likely limited lender modifications. Only in clear cases where the lender could exclude likely strategic defaulters through screens and filters could a modification proceed.\(^{28}\) Second, it offers a rationale for a standard government-supported modification.

\(^{28}\) A perverse example occurred early in the crisis, as pointed out by Mayer, Morrison, Piskorski, and Gupta (2014), when the
contract. That is, if the government supported and subsidized a standardized contract for all modifications, then the unraveling problem disappears.

7.2 Ex ante security design

We have thus far consider the optimal choice of \((t_1, t_2)\), which is a modification of a pre-existing mortgage loan, in a stressed crisis environment. Alternatively, policymakers may consider policies and ex ante contracts that help to avoid the ex post renegotiation and unravelling problems we note above. Hence, we now shift back to a pre-crisis date 0 and examine the optimal state-contingent design of the mortgage loan. Using ex ante policies in principle avoids the moral hazard problems associated with ex post loan modifications, and also the pragmatic problems with swiftly modifying potentially tens of millions of individual contracts in a crisis environment (as emphasized and documented in Agarwal, et al, 2011).

Suppose that at date 1, the economy may be in one of two scenarios: the \(\omega = G\) scenario is one where the household has income of \(y_1^G = y_2^G = \bar{y}\) at both date 1 and date 2; the \(\omega = B\) scenario is one where the household is stressed. Below, we consider a variety of possible stress outcomes in state \(B\). The probability of \(\omega = B\) is \(\pi^B\).

The price of \(c^h\) units of housing at date 2 is \(P^c_2\). The price at date 0 is

\[
P_0 = r c^h + r c^h + E[P^c_2].
\]

A loan contract which allows the household to purchase \(c^h\) housing involves an initial payment of \(P_0\) to the household. In return, the household promises payments of \((l^{\omega}_1, l^{\omega}_2)\). Note that \(l^{\omega}_2\) includes the principal repayment. Loan payments are feasible for the borrower as long as

\[
l^{\omega}_1 \leq y^{\omega}_1, \quad l^{\omega}_2 \leq y^{\omega}_2 + P_2^\omega.
\]

The borrower does not default on a loan payment at date 2 as long as

\[
l_2^\omega \leq \theta + P_2^\omega.
\]

Countrywide modification program was made available to borrowers who defaulted by a future date, inducing strategic default leading up to the specified time. Such a design increased the cost of the program, whereas our model suggests program features to limit this adverse selection problem for modifications.
Since defaults involve a deadweight cost, we impose this as a no-default constraint on the households problem. For the lender, the loan must be profitable

\[ P_0 \leq E[l_1^1 + l_2^2]. \]  

(32)

The borrower solves

\[ \max_{l_1^B, l_2^B, l_1^G, l_2^G} (1 - \pi^B) \left( C_1^G + C_2^G \right) + \pi^B \left( C_1^B + C_2^B \right), \]  

(33)

where

\[ c^h \equiv c_t^{h,G} = c_t^{h,B} \]  

(34)

\[ c_t^{\omega} = y_t - l_t^{\omega} \]  

(35)

\[ c_t^{\omega'} = y_t - l_t^{\omega'} \]  

(36)

and subject to the feasibility, default, and lender profit constraints.

This problem has a few properties worth noting regardless of the stress scenario \( B \). First, since default involves deadweight costs and loan payments can be chosen in a fully state contingent manner, there is no benefit to choosing loan payments that result in ex-post default. Any choice of loan payments that results in ex-post default can be improved on by another choice that avoids default, strictly increasing the payoff to both borrower and lender. Second, because contracts avoid default, it also follows that housing consumption will be constant across all dates and states. We have imposed this directly as \( c_t^{h,G} = c_t^{h,B} \). Third, the borrower’s problem then becomes a standard consumption smoothing problem in which he tries to balance consumption of the housing good and the non-housing good at each date. The choices of \( (l_1^B, l_2^B) \) are made to provide consumption smoothing benefits and allow the borrower to purchase a home of size \( c^h \).

The best \textit{ex ante} mortgage contract depends on what type of shock the household faces at date 1. We consider both income and home price shocks, which may be temporary or permanent.

\subsection*{7.2.1 Temporary date 1 income shock}

We first consider a temporary income shock, and then add a concurrent decline in home prices.

Consider a case where \( y_1^B = y_1 < \bar{y} \) while income at other dates is \( \bar{y} \). Further assume that \( P_2^G = P_2^B \).
The following solution achieves the maximal utility for the borrower:

\[ I_1^G = l, \quad I_2^G = l + P_2, \quad I_2^B = l + P_2, \quad I_1^B = l - (\bar{y} - y_1) \]

The household increases payments in the non-constrained states to reduce payments in the constrained state. The contract can be implemented as a fixed rate mortgage with a somewhat higher coupon than a standard mortgage. In the bad state, \( B \), the contract calls for a reduction in date 1 mortgage payment, but no reduction at date 2. This amounts to payment reduction at date 1 \((t_1 > 0, t_2 = 0\) in our earlier notation\) to ease the borrowing constraint and smooth consumption. This could be achieved by indexing payments to economic outcomes. A natural example would allow the borrower the right to convert his fixed rate mortgage to a floating rate mortgage, since in a typical recession state, the central bank reduces short-term interest rates. This contract currently exists as a mortgage contract with a prepayment option, which allows a borrower to prepay the loan and refinance into a lower rate loan. To maximize payment reduction, this would include refinancing into a floating rate mortgage. During the financial crisis, this option was curtailed for underwater loans, since they could not be refinanced (there was no new loan available for the equivalent amount, since it exceeded the collateral value of the home). We address this issue next.

### 7.2.2 Temporary date 1 income shock and a decline in house prices

Next consider a case where income at date 1 in the \( B \) state is low, and additionally \( P_2^B < P_2^G \). The following solution achieves maximal utility:

\[ I_1^G = l, \quad I_2^G = l + P_2^G, \quad I_2^B = l + P_2^B, \quad I_1^B = l - (\bar{y} - y_1) \]

The solution again calls for payment reduction at date 1. Additionally, the solution calls for principal reduction at date 2 in the \( B \) state, commensurate with the decline in prices in the \( B \) state. The principal reduction avoids the dead-weight costs of default by strategic defaulters, while the payment reduction avoids low consumption at date 1 and dead-weight costs of default among the cash flow constrained. This requires a more complex contract since the payment reduction and the loan writedown are not coincident. However, noting the parallels between refinancing and principal reduction, this contract can also be implemented by an expansion of existing contracts. In particular, a refinancing into an adjustable rate mortgage reduces the date 1 payment, and with the option to refinance even if a loan is underwater, as allowed by the HARP
program, for example, also implements a reduction in the present value of payments and prevents strategic default. In this dimension, it is analytically equivalent to principal reduction, but is spanned by existing contracts. We think of this contract as a housing market version of automatic stabilizers, in that it provides state-contingent support to the housing market. This "Stabilizer Contract" reduces payments when the economy is cyclically weak and liquidity constraints are likely to bind, and also reduces loan value when home prices fall.

7.2.3 Permanent income reduction in $B$ and a decline in prices

Finally, if the income shock is permanent, in addition to the home price decline, we have a case where income in the $B$ state at both dates is $y_1$ and $P^B_2 < P^G_2$. The solution is:

$$l^G_1 = l, \quad l^G_2 = l + P^G_2, \quad l^B_2 = l - (\bar{y} - y_1) + P^B_2, \quad l^B_1 = l - (\bar{y} - y_1)$$

The solution calls for a reduction in payments at both dates 1 and date 2, as well as principal reduction at date 2. This contract can also be implemented with the stabilizing ARM refinancing option, with the refinancing contingency that also applies to underwater loans. The extent to which payments are permanently lower in this contract depends on the term structure of interest rates, if the household remains in an ARM contract, or on the refinancing choice of the household, if they refinance into a fixed rate mortgage. The ARM contract would likely provide more liquidity relief at date 1, but a fixed rate contract could provide more long-term payment reduction, since interest rates would tend to rise over time and be reflected in the payments of the ARM borrower.

7.3 Comments on contracts and modifications

The optimal contract calls for lower payments when liquidity constraints bind and a reduction in mortgage obligations when home prices fall; the former allows for consumption smoothing and the latter avoids strategic default and the associated deadweight costs. Various forms of home price insurance or indexation of contracts to home prices have been proposed (for example, Mian and Sufi (2014)) to address the problems posed by negative equity. If implemented at date 2, before default, these options also implement the intent to avoid strategic default at date 2. Some contracts of this ilk have been implemented on a small scale, though issues
with measuring home prices at the appropriate level of aggregation and allowing for home improvements and maintenance incentives pose some practical issues. Indexing to interest rates, as suggested in the stabilizing contract, has the advantage of observability and consistency, preserving monetary policy effectiveness, and the fact that contracts with this feature already exist and are implemented and priced on large scale.

As we discussed earlier, the parallels between principal reductions and refinancings has been largely unnoticed, perhaps because of the focus on face value in principal reductions. However from both the lenders’ and the borrowers’ points of view, the value/cost of a loan is the present value of payments, which may be equivalently reduced by changing face value or the contract interest rate. For example, the Home Affordable Refinancing Program (HARP), which allowed refinancings of underwater GSE loans, is estimated to have completed 3.1 million HARP refinances through the first quarter of 2014, of the 19.2 million total refinancings at the GSEs over the same period. The HARP refinances include loans with LTV exceeding 80 percent, with about 12% of loans exceeding LTV of 125%. Interestingly, the GSEs started offering shorter-term (15 to 20 year) refinancing alternatives under HARP, and about 20 percent of underwater borrowers (LTV greater than 105) have shortened term in this way when refinancing. Consistent with our characterization of principal reduction for unconstrained underwater borrowers, this suggests that these borrowers are not liquidity constrained: by taking a shorter term mortgage, they increased their mortgage payments when they could have chosen lower payments by extending the term of their new mortgages.29

To get a sense of magnitudes, the 30-year fixed rate loan rate hit a trough in November 2012 at 3.35 percent (Freddie Mac PMMS, monthly average). Its peak in 2008 was 6.48 percent. If we use an average decline of 150 basis points due to refinancing on an average loan balance of $150,000, the present value of payments falls by $28,000. The same payment reduction could have been achieved with a reduction in face value of 16 percent, or $24,000 for this typical loan. This method of achieving debt reduction relies on the sharp reduction in mortgage rates that occurred during the crisis. Empirical work has begun to examine the effectiveness of payment reduction through refinancing, including Fuster and Willen (2013) and Bond, Elul, Garyn-Tal, and Musto (2014), who estimate that refinancing reduces the likelihood of mortgage default in the following year by one-third. Work on other forms of cash transfers, such as Hsu, Matsa, and Melzer (2014) that we noted earlier, suggest that they can be effective in avoiding foreclosures. Separately, as emphasized by John Campbell’s (2006) Presidential Address to the AFA, and more recently by Keys, Pope

and Pope (2014), even households who are not underwater may not refinance when it appears to be available and desirable, so a mechanism to automate refinancing may have other social benefits.
8 Conclusions

The structure developed in this paper is very simple, but is intended to provide a conceptual framework for considering policy responses to a housing crisis and recession. Its important features include liquidity constraints, so that households cannot access housing equity or credit markets to smooth consumption, and the possibility of being underwater, so that households not only have no home equity but may find it preferable to default on their mortgage, even when faced with deadweight costs of default. In this setting, payment reduction during the crisis has favorable properties, both for supporting consumption during the crisis and hence achieving better macroeconomic outcomes, but also in reducing default during the crisis. Principal reduction can be helpful, but is a less efficient use of government resources, since it back-loads payments to households who cannot borrow against these future resources to support consumption today, and also because it is most helpful in reducing strategic default, rather than payment-distress-induced default. Defaults resulting from payment distress have a greater negative impact on home prices, since distressed borrowers carry their distress into the rental market and reduce housing demand more than default resulting from strategic considerations. When addressing strategic default, lender incentives are aligned in the sense that lenders should renegotiate before default in order to avoid credit losses; the loan is worth more to the lender than is the collateral. Nonetheless, under uncertainty it is still optimal for lenders to delay renegotiation as long as possible before default.

The government may take a different view than private agents for various reasons. The government may value consumption and macroeconomic performance more than individual agents, and it may take foreclosure spillovers into account. These considerations should lead the government in two directions: first, it should tend to provide more resources during the crisis period as a countercyclical measure - both to support consumption and avoid default, and second, it should support lender efforts at renegotiation in period 2, either by providing incentives or by providing a standardized way of modifying and writing down loans to avoid strategic default and the associated deadweight costs, since private market efforts may be socially insufficient or may collapse entirely due to adverse selection.

Anticipating these ex post difficulties, an ex ante contract could incorporate a stabilizing contract, through an expanded prepayment option. The standard prepayment option allowed for payment reduction as interest rates were substantially lower during the crisis. In particular, refinancing into a floating rate ARM would
allow for a much lower mortgage payment, easing the consumption constraint. However, when loans are underwater, prepayment is problematic, as borrowers cannot finance the underwater portion of the loan. Hence, an expanded refinance option to allow refinancing into an ARM even when the loan is underwater would implement the optimal contract and fill the role of automatic stabilizers in the housing market. This stabilizing contract gives a state-contingent modification to reduce payments and solves the debt write down problem in the state when home prices fall to the extent that interest rates fall coincidentally.

We have limited consideration to policies around housing, and in particular, around mortgages. Other forms of fiscal and monetary policy may be useful in our setting; indeed, the fact that our proposed mortgage contract is indexed to interest rates suggests that monetary policy is powerful in this setting. Similarly, fiscal policy to transfer resources to date 1 and alleviate the liquidity constraint would be effective, and perhaps more so than a housing payment reduction. Because we are addressing the implications of a housing crisis, which includes falling home prices, however, we focus on housing by design. In particular, policies to reduce mortgage default may have outsized effects in a housing crisis, so focusing resources on mortgage borrowers may be unusually relevant. Moreover, targeting homeowners may be an especially effective way of reaching liquidity constrained households during a housing crisis. This does not say that it is universally more effective than transfers or tax policy to increase liquidity more generally in the crisis period, but as we show, mortgage policies may alleviate distress induced by a home price collapse.

Finally, we have intended this structure to provide a framework for considering various types of credit policy in a simple setting. As credit policy becomes a common component of both fiscal and monetary policy, such a framework may be useful more broadly. For example, empirical questions have arisen around the use of credit policy to finance human capital acquisition (student loans), as well as housing (through the GSEs and FHA), where such a framework could be a valuable tool.
9 References


