Informative Advertising with Discretionary Search

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Abstract

We consider a model of strategic information transmission in which a firm can communicate its quality to consumers through informative advertising. Our main result is that informative advertising claims can be credible even when the firm faces consumers with ex ante homogeneous preferences. A fundamental assumption of our model is that whether the product is available for purchase is independent of consumers’ information acquisition efforts (i.e., search is discretionary). This assumption, in conjunction with the pricing problem of the firm, provides incentives for truth-telling.

When quality is common knowledge, increases in quality lead to a higher market price. However, firm profit and consumer welfare are non-monotonic in product quality. The firm may be worse off with a better product because of increased consumer search and resulting preference heterogeneity. Consumers may also become worse off with a higher quality product when the option value of searching is low because in this case the firm raises price quickly in order to target consumers who do not search. Finally, when product quality is unknown but credible information is available, consumers become worse off with the probability of facing a high type firm because this firm is able to extract value from trade most effectively.

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1 Introduction

The strategic communication between firms and consumers has been the focus of recent work by economists and marketing scientists. The underlying tension behind the phenomenon is the fact that consumers often appear willing to believe advertisements’ claims without verifiability commitments, evidence or proof. This is especially striking because the objective of the sender of such messages is likely to be her own welfare maximization rather than the consumers’. These types of messages are pervasive in product positioning, i.e., in conveying the attributes and qualities that firms would like consumers to associate their products with. Examples of informative messages include diverse elements such as slogans and taglines, property descriptions and posted prices in real estate markets, claims of performance and comfort in automotive markets, etc. Informative advertising also incorporates the fact that sellers are allowed some legal freedom when choosing advertising content. This ability is often defined as ‘puffery’. It protects firms that make claims and exaggerations that are to be reasonably expected by consumers.¹

The credibility paradox has found resolution in the idea that under certain circumstances the firm may have an incentive to publish truthful information in hopes of attracting some preferred segments of consumers rather than others. For example, Bagwell and Ramey (1993) proposed that low and high quality firms could differ in their preferences over segments of consumers. When firm types are significantly different the low end prefers to make small profit margins from multiple units sold to low valuation consumers, while the high end ones prefer to make high margins off high valuation consumers willing to buy only a few units. The mechanism at work is related to a more general principle of informative communication initially proposed by Farrell and Gibbons (1989). It is that the presence of different audiences can introduce a tradeoff to the sender, which ultimately grants her credibility. For example, the reason a firm can successfully attract high type consumers in the work of Bagwell and Ramey (1993) is because its message simultaneously detracts low type consumers from searching. Similarly, a low type firm would prefer to attract the low type consumers at the expense of detracting the high type ones. This logic informs the credibility mechanism proposed by Bagwell and Ramey (1993) in which advertising “matches products to buyers”, which has since remained as the solution to the credibility paradox.

¹For example, puffery is defined by the Federal Trade Commission in the United States of America and by the Advertising Standards Authority in the United Kingdom.
This work uncovers a novel mechanism for the occurrence of credibility in markets, which does not rely on the existence of multiple audiences or of ex ante heterogeneous consumer segments as discussed above. Rather, it is the market interaction, namely consumers’ search decisions and the firm’s strategic pricing response, that enables advertising informativeness and credibility. In the

Figure 1: Contingent Willingness-to-pay with Known and Unknown Product Qualities

current context advertising credibility arises as an outcome of the search intensity by consumers, which derives from allowing them to endogenously decide their information acquisition strategy. The intuition is aided by Figure 1. Consider a product with some known quality $q$, which denotes the probability of a consumer finding that the product fits her needs. In particular, the consumer receives gross utility $v_H$ if she finds a fit, but $v_L$ if she does not, and $v_H > v_L > 0$. If fit is not known a priori the consumer’s ex-ante willingness-to-pay is equal to $E[v] = q v_H + (1 - q) v_L$. However, if she decides to search to learn more about the fit, her valuation will be $v_H$ with probability $q$ and $v_L$ with probability $1 - q$. This is depicted in panel a) of Figure 1 and summarizes a first important effect: After search takes place valuations become more heterogeneous as consumers find whether the product fits their needs or not. This in turn creates a problem for the firm in terms of surplus appropriation: If consumers do not search the firm could capture all of the consumer welfare by setting price at level $p = E[v]$ and earn profit $E[v]$. However, when some consumers search the firm faces a more heterogeneous preference distribution and will become worse off. In the limit it could receive profits $q v_H$ if it set price at level $p = v_H$ or it could receive profits $v_L$ if it set price at level $p = v_L$, both of which are lower.
than the previous case. This first case highlights the fact that as consumers search and find out whether the product fits their needs, their preferences become more heterogeneous, making it harder for the firm to appropriate value.

Consider now the case of unknown product quality, summarized in panel b) of Figure 1. In this example nature has the first move and decides whether the firm has a high or a low quality product, with \( q_H > q_L \) being the respective probabilities of product fit. Consumers do not know which product quality the firm holds, but as before can determine whether the product fits their needs through costly search. When they do not search their expected valuation is given by \( E[v|I] \) where \( I \) stands for the consumers’ beliefs about quality at the first information set. For example, if no advertising information is available to consumers their willingness-to-pay (if they do not search) may be equal to \( E[v|I] = E[v] \).

Credibility arises when the firm has an incentive to report its quality truthfully. This can happen when consumers are more likely to search when they believe the firm is of higher quality and moreover the high and low quality firm types prefer prices close to \( v_H \) and \( v_L \), respectively. By reporting truthfully the low type firm reduces search and avoids the profit losses from the resulting market heterogeneity, as explained above. Moreover, if it imitated the high type firm it would decrease its profits because the additional searching customers would apply downward pressure on the price it is able to charge. The high type firm however, is willing to induce more search because it knows searching consumers are likely to find a fit. Moreover, it can alleviate the heterogeneity concern if these customers are profitable enough, which ultimately depends on how high the quality of its product is.

A fundamental assumption in the present work is that consumers can decide whether to acquire information about the product before making their purchase decision. We define this behavior as ‘search’. As in Nelson (1970), consumers can engage in costly pre-purchase activities to understand a product’s merits before making a purchase decision. We generalize search decisions, however, by allowing consumers to determine themselves whether they would like to engage in search or not, according to their own interest. This is in contrast with the more common and restrictive ‘search good’ assumption in which search is strictly required for purchase.

In our setting the search term is applicable to multiple phenomena. The first interpretation

\(^{2}\)These profits can be thought of as an upper-bound, since at these price levels consumers would be better off not searching when facing a strictly positive search cost.
is that consumers incur a cost to learn about product features and attributes before purchase, be it through online information gathering, open houses, store visits, test drives, etc. A second interpretation is that consumers can engage in deliberation, that is, they may resort to costly thinking to understand whether the product appropriately fits their needs. This mechanism is discussed by Guo and Zhang (2012): Consumers are uncertain about their own preferences but can incur a cost to assess product fit. The reason that in our model search allows alternative interpretations is that no matter the particular focus of the consumer, the fundamental construct informing her decision is whether a fit between her preferences and the product characteristics exists. See also Guo and Wu (2015) for a detailed discussion.

We focus on the case in which consumers can engage in costly search and deliberation for product attributes before purchase. In our model price information is revealed after the search decision has been taken. This is a common assumption in the informative advertising literature. In effect there exist multiple settings where eliciting price information is extremely hard because of negotiation (e.g.: automobile and mattress purchases), hidden fees and charges (e.g.: telecommunications and insurance), or because final price is determined through particular mechanisms (e.g.: auction settings including real estate transactions). Nelson (1970) states that product features can be the primary object of search. See also Dickson and Sawyer (1990), Lynch and Ariely (2000) and Gabaix and Laibson (2006) for related rationales.

In our model the advertising message is cheap-talk in the sense that the choice of the content has no relevant cost implications for the sender (as in Crawford and Sobel, 1982). In the informative advertising literature Bagwell and Ramey (1993) consider the case of firms inducing search by their favorite customer segments through the advertising strategy. Relatedly, Gardete (2013) shows that firms may have an incentive to pool their claims upward but only to a certain extent, and that overall surplus may be maximized when they are allowed to do so. Chakraborty and Harbaugh (2010) and Chakraborty and Harbaugh (2014) consider the case of informative multidimensional advertising. In this case the seller can attain credibility because she sells a multi-attribute product. Credibility can arise if she emphasizes some of the product characteristics, but not all. All of the available work relies on the audience heterogeneity idea proposed by Farrell and Gibbons (1989), which our work dispenses.\footnote{A related literature started by Wernerfelt (1990) considers the role of cheap-talk in advertising as a means to coordinate consumers’ signaling to each other. Recently, Kuksov, Shachar, and Wang (2013) investigate a related case where the firm’s advertising decision is affected by the correlation between consumer preferences for}
It is also worth mentioning the advertising disclosure literature, which also focuses on the content of advertising. In contrast with the one described above, that literature explains cases in which firms are not legally allowed to misrepresent themselves. For example, in the North American automobile market there exists regulation on how mileage per gallon should be calculated and communicated by automobile manufacturers. Failure to abide by these rules can result in lawsuits and fines. In these cases the firm’s communication is endowed with commitment power. For example, Anderson and Renault (2006) and Anderson and Renault (2013) consider the disclosure decision of firms in markets for search goods with horizontal and vertical differentiation, respectively. Guo (2009) and Guo and Zhao (2009) analyze cases of quality disclosure in retail channels and in competition settings, respectively. Anand and Shachar (2009) consider the fact that advertising messages can convey different meanings to different consumers, and the fact that the firm may strategically decide how precise its advertising should be. Finally, Mayzlin and Shin (2011) consider the case in which a firm can affect consumers’ search decisions through its communication in the case of multiple attributes and bandwidth-limited consumers. Related to this paper, Mayzlin and Shin (2011) find that because advertising content can affect search behavior, it can create incentives for informativeness. In our case, however, advertising and consumer search act as complements rather than perfect substitutes. Moreover, in contrast with their paper in particular and with the literature above in general, our model features a number of characteristics that could apparently preclude advertising credibility. Crucially, in our case the firm is allowed to misrepresent itself. In addition, it faces fully strategic consumers (they can correctly understand its communication and are not constrained by cognitive resources), and the firm communicates about a single vertical dimension. Despite these apparently limiting conditions, we find that advertising credibility still emerges because of the market interaction between the firm and consumers.

Finally, a third stream of the advertising literature initiated by Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) focuses on high advertising spending as a signal of high quality. In contrast, this article (as well as those cited above) explains conditions for credibility to emerge because of the content of advertising.


5See Becker and Murphy (1993) for interesting discussions on the scope of this literature, as well as Caves
We now turn to the main analysis. The next section presents the simpler case of a vertically
differentiated market in which product quality is observable by both consumers and the firm. As we describe below, product quality influences the likelihood of finding product fit, i.e., a high quality product is more likely to fit customer needs than a low quality one, ceteris paribus. Section 3 introduces the case of asymmetric information: Consumers do not observe product quality (it is now the firm’s private information) but receive an advertising message that can be taken into consideration in their search and purchase decisions. Finally, Section 4 presents a discussion of the results and their generalizability and Section 5 presents some concluding remarks.

2 Known Product Quality

2.1 Preliminaries

We consider a market for a product with quality \( q \in (0,1) \). Consumers do not derive utility from product quality directly. Instead, \( q \) determines the likelihood of finding product fit. In particular, with probability \( q \) the product yields gross utility \( v_H \) upon consumption and with probability \( 1 - q \) a consumer receives gross utility \( v_L \), where \( v_H > v_L > 0 \). Moreover, we normalize the utility of not buying to zero.

In this section we assume consumers know the product quality while product fit is not known a priori. In order to learn fit consumers may engage in costly search. In general, the search process can be thought of as a costly investment that rewards the customer with product fit information. Consumers can decide whether they would like to engage in costly search or not. Only if they decide to search do they incur a search cost \( c > 0 \) and learn their own fit with the product before buying. A fundamental feature of the model is that consumers can buy regardless of their search decision. Hence, the search decision affects their information but not their ability to buy the product. Figure 2 presents the timing of the game.

First, consumers make their search decisions. The firm then makes a take-it-or-leave-it offer through price \( p \). After this, consumers observe price and those who decided to search also observe their fit with the product. Finally, consumers make their purchase decisions.

and Greene (1996) for empirical tests.
Figure 2: Timing of the Perfect Information Game

Consumers decide whether to search

Firm sets price

Consumers observe price. Fit is revealed to searchers

Consumers make purchase decisions

Keeping the price disclosure to a later stage is a common assumption in the informative advertising literature in order to isolate the informational effects of the advertisement, and avoid imposing beliefs when inconsistent prices and advertising messages occur. This assumption is especially useful in Section 3 in order to focus the results on the informativeness of the advertising message (See Bagwell and Ramey (1993) and Gardete (2013) for discussions). Moreover, note that because price is not observable until search decisions are taken, it can equally be set before consumers make their search decisions, with no effect on outcomes.

2.2 Customer Decisions

Consider the last stage of a customer’s decision process: the purchase decision. At this point the customer has already decided to search or not. If she decided not to search she will buy if

\[ E[u_B] \geq 0 \iff E[v|q] - p \geq 0 \] (1)

where \( E[v|q] = q v_H + (1 - q) v_L \) captures the expected gross utility from purchasing the product. If the consumer has searched, however, the decision to buy depends on whether the product fits her needs or not. In particular, if fit occurs she buys if

\[ v_H - p \geq 0 \] (2)

and if fit did not occur she will still decide to buy as long as

\[ v_L - p \geq 0 \] (3)
Consider now the decision of whether to search or not. At this stage the consumer does not yet observe the product price, but holds a belief $\hat{p}$ about it. She decides to search if

$$E[U(\text{Search})] \geq \max \{E[v|q] - \hat{p}, 0\}$$

(4)

where the left hand-side captures the expected utility of searching and the right hand-side captures the best of the two possible scenarios that the consumer may face if she does not search: She receives the maximum utility between buying the product or not in the absence of product fit information. In particular, if $E[v|q] \geq \hat{p}$ she believes that she will buy the product if she decides not to search, and if $E[v|q] < \hat{p}$ she believes that she will not buy the product if she does not search. We now consider these possibilities in turn.

First, let the belief over price be such that $E[v|q] \geq \hat{p}$. In this case the consumer searches if

$$q(v_H - \hat{p}) - c \geq E[v|q] - \hat{p}$$

(5)

where the left-hand side reflects the expected utility from search, and the right-hand side captures the expected utility of buying without searching. In order to describe the left-hand side one needs to consider the fact that price is bounded below by $v_L$. If the price were any lower the firm could increase its profits by increasing it until $v_L$. If the product does not provide a fit the consumer always receives zero utility either because she decided not to buy the product ($p > v_L$), or the price extracts all her product valuation ($p = v_L$). It follows from (5) that when $E[v|q] \geq \hat{p}$ the consumer decides to search if and only if

$$\hat{p} \geq v_L + \frac{c}{1 - q}$$

(6)

Consider now the case where the consumer believes the product is “expensive”, such that $E[v|q] < \hat{p}$. In this case she compares the utility from search to the utility of not searching and not buying. In particular she decides to search when $q(v_H - \hat{p}) - c \geq 0$, which rearranged becomes:

$$\hat{p} \leq v_H - \frac{c}{q}$$

(7)

Figure 3 plots how the consumer decisions depend on their beliefs over price.
When the belief over price is low enough (when compared to the search cost) consumers are willing to buy without searching. As the belief increases search becomes an attractive option. However, when the belief over price is very high the option value of searching is low, and consumers prefer not to search. The next result follows:

**Proposition 1 (Emergence of search behavior).** *Search emerges only when \( q \) is intermediate. Otherwise, consumers do not search independently of their belief over price. Moreover, the search cost must be low for search to take place.*

Proposition 1 states that when the probability of finding a fit is very high or very low, search is not valuable to consumers as they prefer to either buy without searching or not to buy at all, depending on the price the firm sets. This is equivalent to the search range in Figure 3 disappearing for very low or very high values of \( q \). We present the search quality range \( q \in (q_0, q_1) \) as well as the proof of Proposition 1 in Appendix A.

The second part of the proposition states that the search cost must be small enough vis-à-vis the utility range for search to potentially arise. When this condition is not met search can never take place and the market outcome becomes trivial. We assume the search cost is low \((4c < v_H - v_L)\) for the remainder of the paper. As we will see in the next section, Proposition 1 informs the nature of the market outcome.

### 2.3 Market Outcome

Consider the pricing behavior of the firm contingent on the search decision of customers. According to Proposition 1, when quality is very low or very high - i.e., \( q \in (0, q_0) \cup (q_1, 1) \) -
consumers do not search. In this case the firm is better off charging $E[v|q]$ and consumers buy the product without searching. No profitable deviation for either party exists and a pure strategy equilibrium with price $E[v|q]$ and no search by consumers takes place whenever the probability of fit is either very high or very low. The result that at very low or very high levels of quality $q$ consumers prefer not to search is intuitive: Consumers will not devote much deliberation to the decision of buying a product that a) is does not fit their needs ‘by far’ or b) fits their needs ‘perfectly’. Only in the intermediate case in which there exists enough uncertainty about product fit is there an incentive to engage in costly deliberation. We denote this intermediate region as $q \in [q_0, q_1]$.

Consider the demand curve faced by the firm,

$$D(p) = 1[p \leq v_H] \alpha(q) q + 1[p \leq E[v|q]] (1 - \alpha(q)) + 1[p \leq v_L] \alpha(q) (1 - q)$$

where $\alpha(q)$ denotes the consumers’ probability of searching. The demand curve suggests a set of three potentially optimal prices: $v_L, E[v|q]$ and $v_H$. The firm maximizes its profit by solving the problem

$$\max_p \pi(p|\alpha(q)) = p.D(p|\alpha(q))$$

by taking the search behavior of consumers $\alpha(q)$ into account. At each relevant price level the firm profit is given by

$$\pi(p|\alpha(q)) = \begin{cases} 
\alpha(q) q v_H, & p = v_H \\
[\alpha(q) q + 1 - \alpha(q)] E[v|q], & p = E[v|q] \\
v_L, & p = v_L
\end{cases}$$

At price $v_L$, the lowest possible utility realization, the firm is able to capture the whole market. At price $E[v|q]$ the firm is able to capture consumers who search and find a fit, $\alpha(q) q$, as well as those who do not search, $1 - \alpha(q)$, but are willing to pay their expected valuation. Finally, at price $v_H$ the firm serves only the proportion of consumers who search and find a fit, $\alpha(q) q$. Given a search intensity $\alpha(q)$ it is possible to determine the best response of the firm. The left panel of Figure 4 depicts the firm best-response to search behavior $\alpha(q)$.

When product quality is very low the firm is generally better off charging $v_L$. This makes
intuitive sense, since in this case the firm prefers to target consumers who do not find a fit with the product, as well as all of the non-searchers. As quality increases the best-response by the firm depends on the search behavior: If many consumers search the firm is better off charging price $v_H$ and extract the surplus from consumers who search and find a fit. However, if not many consumers search the firm is better off targeting the non-searchers by pricing at $E[v|q]$.

In order to understand the behavior of consumers it is useful to define $\beta_L$ and $\beta_H$ as the probabilities of the firm setting prices equal to $v_L$ and $v_H$, respectively. We omit the $q$ argument of $\beta_L(q)$ and $\beta_H(q)$ for parsimony. The search decision provides consumers an option value that can be defined as

$$E[U(\text{Search})] - E[U(\sim \text{Search})] = (1 - \beta_L - \beta_H) q (v_H - E[v|q]) - c$$

(10)

The difference is equal to the utility of searching and finding fit while facing price level $E[v|q]$, minus the search cost. The expression above reveals that making customers indifferent between searching and not searching is equivalent to making this utility equal to zero. The reason is relatively subtle: When price is highest ($p = v_H$) search offers no value to consumers because their maximum possible utility will be equal to zero independently of whether they buy or not. When price is lowest ($p = v_L$) consumers are always willing to buy, and so the search decision
does not provide additional value either. Hence, the only difference between the expected utility of searching and not searching occurs when the firm practices price $E[v|q]$, as denoted in expression (10). At this price consumers may be better off searching (when the search cost is low) because search offers them the option value to buy or not depending on whether they find the product satisfies their needs. At price levels $v_L$ and $v_H$ consumers are better off buying without searching, or not searching and not buying altogether, respectively. However, the firm has the opposite pricing incentives: When consumers search it prefers to charge $v_L$ or $v_H$, and when consumers do not search it prefers to charge the expected utility $E[v|q]$. For this reason an equilibrium with search can only arise in mixed strategies.

In appendix B we show that no mixed strategy equilibrium occurs if the firm mixes between prices $\{v_L, v_H\}$, and in this case consumers would have no benefit from searching, as discussed above. Hence, we look for search strategies that make the firm indifferent between prices in $\{v_L, E[v|q]\}$ and in $\{E[v|q], v_H\}$, and look for potential deviations (including to pure strategies). Consumers make the firm indifferent between prices $\{v_L, E[v|q]\}$ as they mix on the OB curve, on the left panel of Figure 4. Moreover, they make the firm indifferent between prices $\{E[v|q], v_H\}$ by searching along curve BC. We denote these search probabilities as $\alpha_L(q)$ and $\alpha_H(q)$, respectively:

\[
\alpha^*_L(q) = \frac{E[v|q] - v_L}{(1 - q) E[v|q]} \quad (12)
\]

The firm also prices in order to make consumers indifferent between searching and not searching. This entails keeping expression (10) equal to zero by mixing price between $\{v_L, E[v|q]\}$ with probabilities $\{\beta_L, 1 - \beta_L\}$ and between $\{v_H, E[v|q]\}$ with probabilities $\{\beta_H, 1 - \beta_H\}$. By comparing the firm profits in each regime one can establish the following result:

**Proposition 2 (Market outcome with known quality).** When quality is low consumers search according to $\alpha^*_L(q)$ and the firm mixes between prices $\{v_L, E[v|q]\}$. When quality is high consumers search with probability $\alpha^*_H(q)$ and the firm mixes between prices $\{v_H, E[v|q]\}$. When quality $q$ is either very high or very low consumers buy without searching and the firm sets price equal to $E[v|q]$. 
The proposition above is technical and is defined precisely in appendix B. However, it is easy to translate to the real world. The last result of no search when quality is either very low or very high has already been discussed. It is worth analyzing the intermediate range. A product of low quality (but high enough to induce search) promises a low probability of fit. Because of this the firm decides to target two types of consumers, namely those who decide not to engage in costly information acquisition as well as those who having decided to visit the store to acquire more information, for example, do not find the highest level of product fit. While some consumers do find product fit, the relatively low likelihood of this event leads the firm not to charge price $v_H$. The same reasoning applies to a high quality product: because most consumers will find fit, the firm is better off not charging $v_L$. Instead it prefers to charge price levels $E[v|q]$ and $v_H$ so as to target consumers who either decided not to search, or having decided to search have found a fit with the product.

The market outcome is characterized by mixed strategies in prices. The most common interpretation of the result is that of price promotions, in part popularized by Varian (1980). In contrast to the idea that firms offer promotions to make each other indifferent between price levels, as in Varian (1980), in our setting a single firm finds it profitable to offer promotions to make sure it is not taken advantage of by customers’ information acquisition strategies. The pricing policy is given by

\[
\Pr (p = v_L) = \begin{cases} 
0 & q > \bar{q} \\
\beta^*(q), & q_0 < q \leq \bar{q} \\
0, & q \leq q_0
\end{cases}
\]  \hspace{1cm} (13)

\[
\Pr (p = v_H) = \begin{cases} 
0, & q > q_1 \\
\beta^*(q), & \bar{q} < q < q_1 \\
0 & q \leq \bar{q}
\end{cases}
\]  \hspace{1cm} (14)

\[
\Pr (p = E[v|q]) = 1 - \beta^*(q)
\]  \hspace{1cm} (15)

where

\[
\beta_L^* = \beta_H^* = \beta^*(q) \equiv 1 - \frac{c}{q(1-q)(v_H - v_L)}
\]  \hspace{1cm} (16)
The pricing policy is shown on the right panel of Figure 4, and \( \bar{q} \) is defined in Appendix B. Along policy \( \beta^*(q) \) consumers are indifferent between searching and not searching if \( q \in (q_0, q_1) \). When \( q < \bar{q} \), \( \beta^*(q) \) denotes the probability that the firm will set price \( p = v_L \) in the mixture \( \{v_L, E[v|q]\} \), and when \( q > \bar{q} \), \( \beta^*(q) \) denotes the probability that the firm will set price \( p = v_H \) in the mixture \( \{v_H, E[v|q]\} \).

The optimal search strategy for consumers is given by

\[
\alpha^*(q) = \begin{cases} 
0 & q > q_1 \\
\alpha_H^*(q), & \bar{q} < q \leq q_1 \\
\alpha_L^*(q), & q_0 < q \leq \bar{q} \\
0, & q \leq q_0
\end{cases}
\]

The search probability in the intermediate quality range \( (q_0, q_1) \) corresponds to the curve OBC on the left panel of Figure 4. We now turn to characterize the market outcome:

**Proposition 3 (Characterization of market outcome with known quality).** The search probability \( \alpha^*(q) \) is either increasing or non-monotonic in quality. Expected price is strictly increasing in quality. Expected profits are increasing in quality except at \( q_0 \), where they decrease. Expected consumer welfare is strictly positive only when \( q \in [q_0, \bar{q}] \). In this region it increases in \( v_H \), it decreases in \( c \) and \( v_L \) and it is non-monotonic in \( q \).

The proof of this Proposition is presented in Appendix C. Here we use Figure 5 to depict the search probability at different quality levels for two different cases. In both cases the search probability is equal to zero for very low and very high levels of quality because the option value of searching is not high enough to compensate the search cost. The left panel depicts the search probability when product fit does not constitute a significant change in utility, i.e., \( v_H < 4v_L \). In the intermediate region \( (q_0, q_1) \) the search probability is strictly increasing in the same range, but it is non-monotonic when \( v_H > 4v_L \) (right panel of Figure 5).

The search strategy can be understood in light of consumers mixing their search decision in order to make the firm indifferent across its actions. When \( q < \bar{q} \) their search behavior makes the firm indifferent between prices \( v_L \) and \( E[v|q] \). Suppose consumers attribute probability \( \bar{\pi} \)
to searching. In this case the firm makes profits

$$\pi(p) = \begin{cases} v_L, & p = v_L \\ E[v|q] (\alpha q + 1 - \alpha), & p = E[v|q] \end{cases}$$

(17)

since if it charges $v_L$ all consumers buy, but if it charges price $E[v|q]$ only consumers who do not search, and those who search and find a fit, will buy. The profit from charging $v_L$ is independent of the search decision, so consumers can only affect the profit of the second decision by changing $\alpha$. Consider what happens to profits when quality increases slightly, to level $q' > q$.

The second branch of profits increases unequivocally both through consumers who search as well as through consumers who do not search. These effects are captured by the terms $E[v|q] \alpha q$ and $E[v|q] (1 - \alpha)$, respectively. Moreover, an increase in quality increases profits from the latter group faster. The reason is that while an increase in quality affects all non-searchers, it only affects some of the searchers (those who find a fit with the product). Thus, by increasing $\alpha$ consumers can reduce $\pi(E[v|q])$ back to level $\pi(v_L)$ because the loss in profits from non-searchers outweighs the gains from searching customers. This explains the positive slope of $\alpha^*(q)$ when $q < \overline{q}$.

When quality is high ($q > \overline{q}$) consumers search to make the firm indifferent between prices

Note: Parameter values used are $v_H = \frac{6}{10}$, $v_L = \frac{1}{5}$, $c = \frac{1}{20}$ and $v_H = 2$, $v_L = \frac{1}{5}$, $c = \frac{1}{10}$ in the left and right panels, respectively.
Consumer search affects the profits of both price levels. For example when search increases, profit \( \pi(v_H) \) increases and profit \( \pi(E[v|q]) \) decreases. However, increases in quality have mixed effects on the difference \( \pi(v_H) - \pi(E[v|q]) \). Hence, it is not at first sight clear how consumers should search along the quality path. The answer lies on whether product fit affects utility to a large extent. When fit does not impact utility too much \((v_H < 4v_L)\) an increase in quality to level \(q'\) yields \( \pi(v_H) < \pi(E[v|q']) \), and so consumers increase search to keep the firm indifferent across its price options. This happens because the firm benefits more from charging \( E[v|q] \) and collect profits from consumers who search and find a fit as well as from non-searchers than from charging \( v_H \) and only collect profits from the first group. In other words, if product fit cannot yield that much more differential utility to consumers who find a fit, the firm benefits from targeting the non-searchers as well.

The same result holds when fit changes the customer satisfaction by a lot \((4v_L < v_H)\), but only when quality is high. When quality is close to \( q \) the firm prefers to target only the consumers who search and find a fit. As quality increases, however, the gains from charging price \( E[v|q] \) increase faster than from charging \( v_H \) because the probability of a consumer finding no fit (and receiving utility \( v_L \)) is significantly decreased. Hence, focusing on consumers who search and find a fit through price \( v_H \) becomes less attractive. This explains the search patterns Figure 5.

The expected price and profits with respect to quality are shown in Figure 6, and they are representative of the general case. The average price increases strictly with quality: When quality is very low the firm is able to extract all surplus. When quality is low \( q \in (q_0, \overline{q}) \) the firm introduces price \( v_L \) in order to target consumers who search but do not find a fit. While the weight on price \( v_L \) increases with quality, it does so at a lower rate than the increase of \( E[v|q] \), and so the average price also increases. At \( \overline{q} \) the probability of fit is high enough that the firm prefers to target consumers who search and find a fit, and introduces price \( v_H \) in the mixture. Finally, above \( q_1 \) search is no longer valuable for consumers, and the firm is able to capture the whole market by pricing \( E[v|q] \).
A striking result is that expected profit is also increasing in $q$ except at level $q_0$. At this level search becomes valuable to consumers and the firm unambiguously loses profits because of the resulting preference heterogeneity. Because the firm can no longer capture the whole market at a single price, at this low level of quality the firm is better off targeting consumers who search and do not find fit. The result that profits decrease at $q_0$ implies that a firm may become worse off with a better product. Because such a product is good enough it attracts interest and consideration. The resulting search intensity leads to ex post preference heterogeneity and results in a value appropriation problem and lower profits for the firm.

The opposite effect takes place at $q_1$: At this level search stops and the firm benefits from reduced preference heterogeneity. This explains the second jump in expected profits, at quality level $q_1$.

Consumers only benefit from quality in range $q \in (q_0, q_\bar{q})$. When $q < q_0$ or $q > q_1$ no search occurs and the firm is able to extract all of the surplus through pricing. Consumers do not receive any surplus when $q \in (q, q_1)$ either, because at this quality level the firm targets consumers who search and find a fit (as well as those who do not search). Hence, the only relevant region in which consumers have strictly positive surplus is when $q \in (q_0, q_\bar{q})$. As expected, in this region consumer welfare increases with $v_H$ and decreases with the search cost. The result that consumer surplus decreases with $v_L$ is related to the fact that in region $q \in (q_0, q_\bar{q})$ the firm mixes between prices $\{v_L, E[v|q]\}$. The first price level targets consumers who search and do not find a fit, and the second targets consumers who do not search. In this region consumers expect surplus
Note: Parameter values used are $v_H = 2$, $v_L = 1$, $c = 1/10$ and $v_H = 3/2$, $v_L = 1$, $c = 1/10$ in the left and right panels, respectively.

$\beta (E [v|q] - v_L)$, because of the proportion of times they find price equal to $v_L$. However, as $v_L$ increases so does the average price, and the difference between prices $\{v_L, E [v|q]\}$ also naturally decreases. These two effects lead to a reduction in consumer surplus.

Finally, the left panel of Figure 7 depicts the case in which consumer welfare is increasing in $q$ in region $q \in (q_0, \bar{q})$. When $v_H - v_L$ is high compared to the search cost welfare is always increasing in $q$ in the present region. The reason is that search provides a high option value, and so the firm prefers to increase the probability of price level $E [v|q]$ slowly in this case and target searching customers instead. However, when the difference in product valuations is low compared to the search cost, search is relatively unattractive in region $q \in (q_0, \bar{q})$. In this case, as quality increases the firm prefers to increase the probability of price $E [v|q]$ faster to target consumers who do not search, and welfare decreases with quality at high values of $q$. This effect results on the inverse-U trajectory depicted on the right panel of Figure 7.

The case with private information and communication over quality is explained next, and it follows from the market characterized above.
3 Unknown Product Quality

3.1 Preliminaries

We investigate the case in which information about product quality is asymmetric. In this case the firm’s quality \( q \in Q \equiv \{q_L, q_H\} > 0 \) is its own private information.\(^6\) In order to communicate its quality the firm can engage in informative advertising by sending a message \( m \in Q \). The content of the message is cheap-talk in the sense that neither the cost of advertising nor the cost of the advertising content vary across firm types.\(^7\) While consumers can use advertising to inform their search decision, they still take the incentives of the firm of sending a given message into account. In particular they understand that a firm may have an incentive for misrepresentation. After incorporating the information asymmetry and cheap-talk advertising, the timing of the model becomes:

![Figure 8: Timing of the Imperfect Information Game](image)

The firm is first endowed by nature with a product of quality \( q \). After it observes its product quality it sends a message to consumers, which can be used to inform their search decision. The rest of the game proceeds as before.

Our interest lies on perfect Bayesian equilibria (PBE) in which advertising is informative such that the firm types effectively use advertising to credibly convey their quality to consumers (i.e., separating equilibria). Hence, consumers are not naive but instead form beliefs on the equilibrium path according to Bayes rule. In contrast with most related work we show there exists only one such informative equilibrium. Moreover, in the only informative equilibrium

\(^{6}\)The results in this section do not depend on the probability of the firm being a high or a low type. In the next section we denote the probability that the firm is endowed with a high quality product by \( \lambda \).

\(^{7}\)This ensures that the outcome is explained purely by informational reasons rather than through alternative costly signaling mechanisms. The fact that the production cost is normalized to zero for both firm types provides the same assurance.
advertising is informative while price bears no signaling power. We formalize consumer beliefs and describe signaling power in Appendix D. We reaffirm that the result relies in part on price knowledge following search decisions and that it does not extend to contexts where final price information is easily obtainable, as already discussed in Section 1.

3.2 Market Outcome

When quality is not observable consumers use their beliefs to decide whether to search or not. The probability of search $\alpha(\tilde{q})$ is the same as before but now is a function of the consumers’ beliefs instead of observable product quality. We are interested in the case where the firm is willing to report its type truthfully. This is captured by the following pair of incentive compatibility constraints

\begin{align}
 IC_1 : \pi^*(L|\tilde{L}) &\geq \max_p \pi(L|H) \\
 IC_2 : \pi^*(H|\tilde{H}) &\geq \max_p \pi(H|\tilde{L})
\end{align}

where $\pi(L|H)$ denotes a low quality firm’s profit as a function of price when it is believed to be of quality $H$, and $\pi^*(L|\tilde{L})$ is the firm’s profit at the equilibrium price(s). Moreover, in equilibrium consumers are willing to believe the claim of the firm, i.e., $b(m) = m$.

**Proposition 4 (Market outcome with unknown quality).** Credible advertising can only take place in region $q_L \in (0, \tilde{q})$, $q_H \in (\tilde{q}, q_1)$. In sub-region $q_L < q_0$ credibility arises when $\frac{v_H - v_L}{v_L}$ is low, or if $\frac{v_H - v_L}{v_L}$ is high when $q_L$ is also high. When $q_L \in (q_0, \tilde{q})$ credibility arises when $q_L$ is low. In both cases $q_H$ is required to be high.

The shaded area in Figure 9 denotes the region where truthful advertising may emerge, $q_L \in (0, \tilde{q})$ and $q_H \in (\tilde{q}, q_1)$. The letters in the remaining regions identify the firm that benefits from misrepresentation.

For example, in the bottom left region the low quality firm has always an incentive to overstate its quality. Because no search takes place in this region, customers are willing to pay their ex-ante expected valuation for the product. If they believe the message of the firm, the low

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8A proof for Proposition 4 is presented in Appendix D.
Figure 9: Incentives to Provide Informative Advertising

Note: Parameter values used are $v_H = 1$, $v_L = 1/5$, $c = 1/10$.

type firm is better off overstating its quality and receiving profits $E[v|q_H]$ rather than $E[v|q_L]$. Above this area we find that the high type firm has an incentive to understate its quality. In this region the quality of the high type firm is high but not high enough that it wants to extract profits mainly from those customers who search and find a fit with the product. Hence, if believed the high type firm is better off imitating the low type firm in order to reduce search or eliminate it altogether.

On the top region ($q_H > q_L$) the low type firm has always an incentive to imitate the high type one. By doing so it could earn profit $E[v|q_H]$ if believed. This is more than it can ever earn if it communicates its type truthfully. The last region where credibility cannot occur is when $\bar{q} < q_L < q_H < q_1$. In this region firms mix in the price support $\{E[v|q_j],v_H\}$, and consumers search with some probability. Because in this region the firms target consumers who search and find a fit, the low type firm is always better off imitating the high type firm in order to extract rents from more searching customers, or from those who did not search but are now willing to pay $E[v|q_H]$ rather than $E[v|q_L]$. The high type firm may also want to imitate the low type firm if this means an increase in the search probability.
In sum, the emergence of credibility requires a set of favorable properties. Intuitively, if consumers are certain that no matter the product quality their needs are likely to be similarly satisfied then communication breaks down. In this case there exists a firm type that finds it ‘cheap’ to misrepresent itself. As we explain next, only if quality exerts enough influence on product valuations can advertising credibility arise.

Figure 10: Incentives to Provide Informative Advertising, \((q_L < q_0)\)

\[
\frac{v_H - v_L}{v_L} < v' \quad \text{and} \quad \frac{v_H - v_L}{v_L} > v'
\]

Note: Parameter values used are \(v_H = 1, v_L = 1/5, c = 1/10\) and \(v_H = 3/2, v_L = 1/5, c = 1/10\) in the left and right columns, respectively.

Consider the sub-region \(\{q_L \in (0, q_0), q_H \in (q_H, q_1)\}\) in which credibility can emerge. In this case the low type firm earns profit \(\pi^*(L|L) = E[v|q_L]\). If believed, by imitating the high type firm it can earn \(\pi(L|H) = E[v|q_H] \left(1 - (1 - q_L) \alpha_H(q_H)\right)\) instead. In this case its margin increases from \(E[v|q_L]\) to \(E[v|q_H]\), but demand drops from 1 to \(1 - (1 - q_L) \alpha_H(q_H)\) because by claiming to be a high type the firm induces search. Whether the tradeoff is appealing depends on the product valuations and the firms’ quality levels. First, note that we can rewrite the gain
in margin from misrepresentation as a function of \( \frac{v_H - v_L}{v_L} \):

\[
\frac{E[v|q_H]}{E[v|q_L]} = \frac{v_L + q_H (v_H - v_L)}{v_L + q_L (v_H - v_L)} = \frac{1 + q_H \frac{v_H - v_L}{v_L}}{1 + q_L \frac{v_H - v_L}{v_L}}.
\]

(21)

and as well as the loss in demand:

\[
1 - \left( 1 - q_L \right) \alpha^*_H(q_H) = 1 - \frac{1 - q_L}{2 - q_H} \frac{(v_L + q_H (v_H - v_L))}{v_L + q_H (v_H - v_L) + v_L} = 1 - \frac{1 - q_L}{1 + (2 - q_H) q_H \frac{v_H - v_L}{v_L}}.
\]

(22)

It is easy to verify that both the gain in margin and the loss in demand from misrepresentation are increasing in \( \frac{v_H - v_L}{v_L} \), and that the first is steeper than the second. When \( \frac{v_H - v_L}{v_L} \) is low the firm does not gain enough from misrepresentation in terms of margin to compensate the loss in demand induced by search. When ratio \( \frac{v_H - v_L}{v_L} \) is high the benefit from increased margins offsets the loss in search as long as \( q_L \) is low. In this case the firm is better off overstating its type in order to receive higher margins from less customers. These cases are depicted in the first row of Figure 10. In the first case (top-left cell) misrepresentation is never attractive, but in the second case (top-right cell) misrepresentation is attractive as long as \( q_L \) is low.

The high type firm faces a simpler problem. By revealing its type truthfully it earns \( \pi^* \left( H | \widehat{H} \right) = \alpha^*_H(q_H) q_H v_H \), and by imitating the low type firm it receives \( \pi \left( H | \widehat{L} \right) = E[v|q_L] \).

Because \( \pi^* \left( H | \widehat{H} \right) \) is increasing in \( q_H \) the high type firm prefers to advertise truthfully as long as it quality is high enough.

These forces originate the parameter spaces depicted in the second row of Figure 10, where the letters denote the firm with an incentive to misreport, as before. In the first case (bottom-left cell) the low type firm never has an incentive to misreport because \( \frac{v_H - v_L}{v_L} \) is low. However, in the second case when \( \frac{v_H - v_L}{v_L} \) is high (bottom-right cell) \( q_L \) must be sufficiently high for credibility to arise. The shape of area \( L \) in the bottom-right cell derives from the fact that \( \pi \left( L | \widehat{H} \right) \) is concave in \( q_H \). The high type firm prefers to report truthfully as long as its quality is high enough, as depicted in the bottom row of Figure 10. Otherwise the firm would be better off imitating the low type firm in order to reduce preference heterogeneity and increase profits. In the appendix we show that in this sub-region firms never have a simultaneous incentive for misrepresentation, i.e., the regions where each firm prefers to misrepresent itself do not intersect.

Credibility can also emerge when \( \{q_L \in (q_0, \overline{q}), q_H \in (\underline{q}, q_1)\} \). Unlike in the previous case,
in this region $q_L$ is high enough to induce some degree of search by consumers. The profit of the low type firm under truthful advertising is $\pi^* \left( L \mid \hat{L} \right) = v_L$ because consumers search to make the low type firm indifferent between prices $v_L$ and $E[v | q_L]$. In this region the low type firm’s profits are independent of its quality. If the firm overstates its quality it receives profits $\pi \left( L \mid \hat{H} \right) = E[v \mid q_H] (1 - (1 - q_L) \alpha^*_H(q_H))$ which is increasing in $q_L$ due to the increased fit of searching customers. Hence, the low type firm is willing to advertise truthfully as long as its quality is not too high.

The high type firm profits $\pi^* \left( H \mid \hat{H} \right) = \alpha^*_H(q_H) q_H v_H = E[v \mid q_H] (1 - (1 - q_H) \alpha^*_H(q_H))$ when advertising truthfully. By charging $p = v_H$ the high type firm can always profit from deviating as long as it can induce more search, since in this case it earns $\pi \left( H \mid \hat{L} \right) = \alpha^*_L(q_L) q_H v_H$. When $\alpha^*_L(q_L) < \alpha^*_H(q_H)$, however, its best deviating option is to charge $p = E[v \mid q_L]$ and earn $\pi \left( H \mid \hat{L} \right) = E[v \mid q_L] (1 - (1 - q_H) \alpha^*_L(q_L))$. The deviation is profitable for low levels of $\frac{v_H - v_L}{v_L}$. The loss in margin and the gain in demand from misrepresentation both decrease with $\frac{v_H - v_L}{v_L}$, although demand increases at a faster rate. When $\frac{v_H - v_L}{v_L}$ is low the demand increase from imitating the low type firm does not compensate the loss in margin, and the high type firm is better off advertising truthfully.

Figure 11: Incentives to Provide Informative Advertising ($q_0 < q_L < q_H$)

![Figure 11](image)

Note: Parameter values used are $v_H = 7/10$, $v_L = 11/50$, $c = 1/10$.

Figure 11 provides an illustration of the regions of interest at a specific set of parameter values. The left panel depicts the fact that the low type firm is better off imitating the high type firm as long as its quality is high enough. The right panel depicts the truth-telling region in the $\{q_L, q_H\}$ space, and the letters identify the company that is better off deviating in a particular
region.\(^9\) The low type firm is better off advertising truthfully as long as \(q_L\) is lower than the upper bound of the shaded region on the right panel of Figure 11. The high type firm is better off advertising truthfully as long as \(\frac{v_H-v_L}{v_L} > v''\). Threshold \(v''\) is decreasing in \(q_H\), such that the high quality firm has a higher incentive to advertise truthfully at higher values of \(q_H\). This translates into \(q_H\) being above the lower bound of the shaded region in the same panel.

Unlike in the previous region, in this case firms may have a simultaneous incentive to misreport their types. It suffices that \(\alpha_H^* (q_H) < \alpha_L^* (q_L)\). If believed the high type firm can increase profits by deviating and charging \(p = v_H\). The low type firm can also increase profits in this case by charging \(p = E[v|q_H]\) and earning \(\pi \left(L|H\right) = E[v|q_H] (1 - (1 - q_L) \alpha_H^* (q_H))\), which is trivially higher than what she can earn by communicating truthfully.

In summary, advertising credibility can only arise when \(q_H \in (\bar{q}, q_1)\). Two cases exist in this region, \(q_L < q_0\) and \(q_L \in (q_0, \bar{q})\). In the first case credibility arises whenever \(\frac{v_H-v_L}{v_L}\) is low or, if \(\frac{v_H-v_L}{v_L}\) is high \(q_L\) is high enough. Moreover, \(q_H\) must also be high for the high type firm not to have an incentive to understate its product quality. Otherwise, by understating its quality the high type firm can deter search and increase profits. When \(q_L \in (q_0, \bar{q})\) credibility is supported by a low level of \(q_L\) such that the low type firm has no incentive to overstate its quality because of lost customers due to increased search. The high type firm will advertise truthfully as long as \(\frac{v_H-v_L}{v_L}\) is high, in which case a deviation would lower margins too much to be attractive. Equivalently, \(q_H\) must be high for credibility to take place.

We now characterize the market outcome:

**Proposition 5 (Characterization of market outcome with informative advertising).** Under truthful advertising the ex-ante price is strictly increasing in both quality levels. Ex-ante profit is strictly increasing in both quality levels, except at \(q_L = q_0\), where it is decreasing. Ex-ante price and profits both increase with the probability of the firm being the high type but consumer surplus is either unaltered or decreases.

The result follows from Proposition 3 because the informative cases are a linear combination of cases with known product quality.\(^{10}\) Let \(\lambda\) denote the probability of the firm selling the high

\(^9\)The truth-telling region can be made much smaller or larger depending on the specific parameters used.

\(^{10}\)The proof is presented in Appendix E.
quality product. In the informative regions the high quality firm always charges more and earns higher profits than the low quality firm, and so it follows that ex-ante price and profits are increasing in $\lambda$. The results for consumer surplus also follow from Proposition 3. A surprising result is that consumer surplus may decrease with $\lambda$, in particular when $q_L \in (q_0, \overline{q})$, because consumers can only earn positive surplus when they face the low type firm. Hence, consumers become (weakly) worse off as the probability of them facing the high quality firm increases.

4 Discussion

In this section we discuss two central features of our model, namely the emergence of mixed strategies and preference dispersion induced by search.

The mixed strategy outcome is related to the Diamond paradox, i.e., the fact that in a market with strictly positive search costs the firms’ optimal strategy is to charge monopoly price, ultimately discouraging search and trade activity altogether (see Diamond (1971); see also Iyer and Kuksov (2012) for a detailed discussion). The argument is usually applicable to settings where consumers are required to incur a search cost in order to learn the price and ultimately buy. However, in our case consumers are allowed to buy without incurring the search cost and the resulting search intensity may generate ex post willingness-to-pay dispersion from ex ante homogeneous consumers.

We now show that the mixed strategy outcome does not depend the discrete/continuous nature of utility. Consider a set of consumers (indexed by $i$) who decide whether to search a product with (known) quality $q$. By searching consumers can learn their gross valuation $v_i = q + \varepsilon_i$, where $\varepsilon_i$ is uniformly distributed with support $[-\gamma, \gamma]$ and is independent across consumers. Consumer $i$’s ex-ante utility from searching is given by

$$E[U(\text{Search})] = Pr(\text{Buy}) E[U|\text{Buy}] - c = Pr(v_i > \hat{p}) E[v_i - \hat{p}| v_i > \hat{p}] - c = \frac{(\gamma + q - \hat{p})^2}{4\gamma} - c$$

As before, consumers may also decide not to search, from which they receive expected utility

\footnote{When $q_L < q_0$ the firm is able to extract all of the surplus, and so $\lambda$ has no effect.}
equal to \( \max \{ E(v) - \hat{p}, 0 \} \). Gardete (2013) shows that an outcome with search in pure strategies exists whenever \( 3\gamma \geq q \), i.e., when uncertainty is high enough relative to the deterministic component of utility. We show that when product uncertainty is less important \( (3\gamma < q) \) our outcome with mixed strategies emerges, even in the case of continuous utility, and credibility is maintained. When \( 3\gamma < q \), consumers prefer to search if \( \hat{p} \in \left[ q - \gamma + 2\sqrt{c\gamma}, q + \gamma - 2\sqrt{c\gamma} \right] \); however, the firm would then be better off pricing at \( p = q - \gamma \), at which point consumers would rather buy immediately without searching. The latter scenario is not an equilibrium outcome because when consumers buy without searching the firm prefers to charge \( p = q \), at which point consumers would be better off searching before buying. It is clear that our result advances the findings in Gardete (2013) by showing that credibility can occur even if uncertainty is not large.

Another fundamental feature of our model is that search induces dispersion in consumer preferences, which makes firms eager to discourage it in some conditions. This result is quite general as we now illustrate through an informal and a formal example. Consider an individual who has become interested in photography, and is considering buying a new camera. Being a novice she is uncertain about the model she should opt for, since cameras are extremely diverse and cater to different needs. Until she knows the extent of her liking for photography and what future opportunities may arise she is left to consider multiple scenarios and their likelihoods. She may later discover she loves photography to the extent of pursuing it as a career, or instead that occasional picture taking will satisfy her. When no more information is available, the consumer’s willingness-to-pay depends on her assessment of the potential scenarios, their probabilities and her liking of each one. In this situation consumers may decide to take a photography course or procure further information to understand their preferences and evaluate the likelihood of the scenarios better. Some consumers will discover they love photography. In their case willingness-to-pay increases after information acquisition while others’ willingnesses-to-pay may decrease or stay relatively unchanged. These consumers had a relatively similar prior before search because they integrated over a number of different scenarios ex ante. After search however, different scenarios become important for different individuals and diverse preferences emerge in the market.

This result can hold even when preference heterogeneity precedes information acquisition. To see this consider the case in which a consumer’s true utility for a product is \( u_i = q + \varepsilon_i - p \), where \( \varepsilon_i \sim N(0, \sigma^2_{\varepsilon}) \). Consumers do not know shock \( \varepsilon_i \), but they can learn it through costly
search. If they decide to search they learn their true shock $\varepsilon_i$. In this case the variance of their willingness-to-pay (“preference heterogeneity”) is equal to $\sigma^2_\varepsilon$. Before searching consumers do not know their preference shock, but may already have heterogeneous preferences. For example, they may have received some information (through conversations with friends, for example) about how the product may fit their needs. Let this information be signal $s_i = \varepsilon_i + \eta_i, \eta_i \sim N(0, \sigma^2_\eta)$. Given this signal, their willingness-to-pay is equal to $q + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta} s_i$, which depends on the specific information each consumer received, $s_i$. If they opt not to search the variance of their gross utility is equal to $V(q + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta} s_i) = \left(\frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta}\right)^2 V(s_i) = \frac{\sigma^4_\varepsilon}{\sigma^2_\varepsilon + \sigma^2_\eta}$. This variance is always lower than the variance of willingness-to-pay after search, $\sigma^2_\varepsilon$, because search ‘collapses’ the expectation of term $\varepsilon_i$ into one realization, leading to a higher heterogeneity in willingness-to-pay.

5 Conclusion

We have considered an asymmetric information model of strategic communication between a firm and consumers. The firm decides the content of its communication and consumers decide whether to believe her claims. Our main result is that informative advertising claims can be credible even when the firm faces consumers with ex ante homogeneous preferences. A fundamental assumption is that whether the product is available for purchase is independent of consumers’ information acquisition efforts (i.e., search is discretionary). At the core of the credibility result is the fact that the resulting search intensity can induce a value appropriation problem for a firm trying to misrepresent itself.

In our model a mixed strategy outcome arises because search provides an attractive option value to customers whenever quality lies in an intermediate range. While the average price increases with quality, the search intensity may be monotonic whenever finding a fit affects utility significantly. In this case the firm prefers to target consumers who search and find a fit when quality is intermediate, locally counteracting the otherwise positive effect of quality on search.

A surprising result is that the firm can become worse off when it holds a better product. This occurs when product quality is just high enough to generate information acquisition incentives for consumers. In this region the firm would have been better off with a lower quality product (albeit selling at a lower price) because in that case search would provide a lower option value.
to consumers. Another unexpected result is that more quality is not always good news for consumers either. Unless quality lies on an intermediate range the firm is able to extract all of the consumer surplus. In the intermediate range more quality can lead to lower consumer welfare if the firm has an incentive to increase prices quickly. This occurs when the incentives to search are relatively low, in which case the firm becomes better off targeting consumers who do not search more as quality increases. Finally, consumers are also worse off if they receive a higher utility in case they do not find a fit with the product because increases in this payoff generate a price increase and further decrease the option value of searching.

Credibility arises when the low quality level is lower than the high quality one, but not by too much. In particular $q_L$ needs to lie in a low range and $q_H$ in an intermediate one. When either quality level is extreme one of the firms has an incentive to misrepresent itself in order to reduce heterogeneity or increase profits from consumers who search and find that the product fits their needs.

This model has investigated strategic information transmission in a static monopoly setting, without the need to include the complexities inherent to reputation and competition settings in order to establish a credibility result. While these two forces have been largely underexplored in the literature they may offer novel insights to the existing knowledge on strategic information transmission.
A Proof of Proposition 1 (Effect of quality on search behavior)

Figure 3 shows that search takes place as long as \( \hat{p} \in [v_L + \frac{c}{1-q}, v_H - \frac{\epsilon}{q}] \). The interval is non-empty as long as

\[
v_H - \frac{\epsilon}{q} \geq v_L + \frac{c}{1-q}
\]

\[\Leftrightarrow q \in \left[ \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4c}{v_H - v_L}} \right), \frac{1}{2} \left( 1 + \sqrt{1 - \frac{4c}{v_H - v_L}} \right) \right]
\]

where the condition \( 4c < v_H - v_L \) is necessary for search to potentially take place. Denote the lower and upper bounds by \( q_0 \) and \( q_1 \), respectively. The search cost condition can be understood by rearranging the terms in (23) to yield

\[
v_H - v_L > \frac{c}{q(1-q)}
\]

Note that the quality level \( q \) that minimizes the right hand side is \( q = \frac{1}{2} \). It is clear that if the condition is not met at the minimizer it cannot be met at any other quality level.

B Proof of Proposition 2 (Market outcome with known quality)

Given the demand structures, the firm can mix among \( \{v_L, E[v|q], v_H\} \). However, given that consumers mix over two actions a good initial guess checks the case where the firm also mixes across two actions.
B.1 Price mixing in \( \{v_L, E[v|q]\} \)

Let \( \beta_L \) be the probability the firm charges \( v_L \) for the product. The firm makes consumers indifferent between searching and not searching, i.e.,

\[
E[U(\text{Search})] = E[U(\sim \text{Search})]
\]
\[\Leftrightarrow \beta_L q (v_H - v_L) + (1 - \beta_L) q (v_H - E[v|q]) - c = \beta_L (E[v|q] - v_L) \]
\[\Leftrightarrow \beta_L^* = 1 - \frac{c}{q(1-q)(v_H - v_L)} \tag{26}\]

B.2 Price mixing in \( \{E[v|q], v_H\} \)

Let \( \beta_H \) be the probability the firm charges \( v_H \) for the product. The firm makes consumers indifferent between searching and not searching, i.e.,

\[
E[U(\text{Search})] = E[U(\sim \text{Search})]
\]
\[\Leftrightarrow (1 - \beta_H) q (v_H - E[v|q]) - c = 0 \]
\[\Leftrightarrow \beta_H^* = 1 - \frac{c}{q(1-q)(v_H - v_L)} \tag{27}\]

B.3 Price Mixing in \( \{v_L, v_H\} \)

Let \( \beta_H' \) be the probability the firm charges \( v_H \) for the product. The firm makes consumers indifferent between searching and not searching, i.e.,

\[
E[U(\text{Search})] = E[U(\sim \text{Search})]
\]
\[\Leftrightarrow (1 - \beta_H') q (v_H - v_L) - c = (1 - \beta_H') (E[v|q] - v_L) \]

which is only satisfied if \( c = 0 \). For a strictly positive search cost the firm cannot make the customers indifferent between searching and not searching in equilibrium.
The two possible outcomes involve mixing within \( \{ v_L, E[v|q] \} \) or within \( \{ E[v|q], v_H \} \). In order to check the conditions under which each mixing strategy dominates (and whether profitable deviations exist), we now consider the behavior of consumers.

### B.4 Consumer Behavior

Suppose consumers search with some probability \( \alpha^*_H \) when the firm mixes in \( \{ E[v|q], v_H \} \), and search with probability \( \alpha^*_L \) when the firm mixes in \( \{ v_L, E[v|q] \} \). The consumers’ optimal strategy is to make the firm indifferent between its prices. Hence, when the firm mixes in \( \{ E[v|q], v_H \} \) consumers search such that

\[
\pi ( p = v_H | \alpha_H ) = \pi ( p = E[v|q] | \alpha_H ) \nonumber
\]

\[
\Leftrightarrow \quad \alpha_H q v_H = E[v|q] (\alpha_H q + 1 - \alpha_H) \nonumber
\]

\[
\Leftrightarrow \quad \alpha^*_H = \frac{E[v|q]}{q v_H + (1 - q) E[v|q]} \quad (28)
\]

Similarly, if the firm mixes prices in \( \{ v_L, E[v|q] \} \) the consumers’ best-response is to search such that

\[
\pi ( p = v_L | \alpha_L ) = \pi ( p = E[v|q] | \alpha_L ) \nonumber
\]

\[
\Leftrightarrow \quad v_L = E[v|q] (\alpha^*_L q + 1 - \alpha^*_L) \nonumber
\]

\[
\Leftrightarrow \quad \alpha^*_L = \frac{E[v|q] - v_L}{(1 - q) E[v|q]} \quad (29)
\]

### B.5 Market Outcome

When consumers search with probability \( \alpha^*_L \) in response to the firm mixing in \( \{ v_L, E[v|q] \} \), no profitable deviations exist for the firm as long as

\[
\max_p \pi ( p | \alpha^*_L ) \leq \pi ( \{ v_L, E[v|q] \} | \alpha^*_L ) \quad (30)
\]
where $\pi \left( \{v_L, E[v|q]\} | \alpha^*_L \right)$ is the firm’s profit with mixing as above, and $\max_p \pi \left( p | \alpha^*_H \right)$ is the optimal deviation. The only candidate deviation is $p = v_H$ (notice that the firm is already indifferent between pricing $v_L$ and $E[v|q]$). Consider the firm profits of charging each possible price:

$$
\pi \left( p = v_L | \alpha^*_L \right) = v_L \\
\pi \left( p = E[v|q] | \alpha^*_L \right) = E[v|q] \left( \alpha^*_L q + 1 - \alpha^*_L \right) \\
\pi \left( p = v_H | \alpha^*_L \right) = \alpha^*_L q v_H \\
$$

Equilibrium requires that

$$
\pi \left( p = v_H | \alpha^*_L \right) \leq \pi \left( p = v_L | \alpha^*_L \right) = \pi \left( p = E[v|q] | \alpha^*_L \right) \\
\leftrightarrow \quad \alpha^*_L q v_H \leq v_L \\
\leftrightarrow \quad q \leq \frac{v_L E[v|q]}{E[v|q] (v_H + v_L) - v_H v_L}
$$

and by expanding $E[v|q]$ we get

$$
q \leq \frac{2v_L}{v_H (5v_H - 4v_L) - (v_H - 2v_L)} \quad (31)
$$

Now consider the case where the firm mixes in $\{E[v|q], v_H\}$ and consumers search with probability $\alpha^*_H$. Consider the firm profits of charging each possible price:

$$
\pi \left( p = v_L | \alpha^*_H \right) = v_L \\
\pi \left( p = E[v|q] | \alpha^*_H \right) = E[v|q] \left( \alpha^*_H q + 1 - \alpha^*_H \right) \\
\pi \left( p = v_H | \alpha^*_H \right) = \alpha^*_H q v_H \\
$$

The firm is better off mixing in $\{E[v|q], v_H\}$ if

$$
\pi \left( p = v_L | \alpha^*_H \right) \leq \pi \left( p = v_H | \alpha^*_H \right) = \pi \left( p = E[v|q] | \alpha^*_H \right) \\
\leftrightarrow \quad v_L \leq \alpha^*_H q v_H \\
\leftrightarrow \quad q \geq \frac{v_L E[v|q]}{E[v|q] (v_H + v_L) - v_H v_L}
$$

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and by expanding $E[v|q]$ we get $q \geq \bar{q}$.

In the text we have also discussed that when $q \in (0,q_0) \cup (q_1,1)$ consumers do not search, and the firm charges $E[v|q]$. This concludes the proof of Proposition 2.

Note that the condition $q_0 < \bar{q} < q_1$ only holds for low enough search cost.\(^\text{12}\)

**B.6 Parameter Regions for Figure 4:**

The profit maximization strategy can be recovered by inspection. It is useful to denote $\alpha^i_j(q)$ as the search probability that makes the firm indifferent between practicing price $i$ and $j$. For example, $\alpha_{vL,E[v|q]}(q)$ denotes the probability of searching that makes firm of quality $q$ indifferent between charging $v_L$ and $E[v|q]$.\(^\text{13}\)

Inspection of the firm’s profits at each price level reveals the following pricing solution:

$$p^* = \begin{cases} v_H, & (q \leq \bar{q} \land \alpha(q) \geq \alpha_{vL,vH}) \lor (q > \bar{q} \land \alpha(q) \geq \alpha_{E[v|q],vH}) \\ E[v,q], & (q \leq \bar{q} \land \alpha(q) \leq \alpha_{vL,E[v|q]}) \lor (q > \bar{q} \land \alpha(q) \leq \alpha_{E[v|q],vH}) \\ v_L, & q < \bar{q} \land \alpha_{vL,E[v|q]} \leq \alpha(q) \leq \alpha_{vL,vH} \end{cases}$$

where $\bar{q} = \frac{2v_L}{\sqrt{v_H^2 - 4v_L}}$. For example, the firm prefers to price at $v_H$ if and only if

$$\alpha(q)qv_H \geq E[v|q](1 - (1 - q)\alpha(q)) \quad (32)$$

and

$$\alpha(q)qv_H \geq v_L \quad (33)$$

The firm prefers to price at $v_H$ if search is high enough. In particular, when its quality is low $(q \leq \bar{q})$ it prefers price $v_H$ if the search probability is higher than $\alpha_{vL,vH}$, but when $(q > \bar{q})$ it requires a search level above $\alpha_{E[v|q],vH}$. Clearly, quality level $\bar{q}$ makes $\alpha_{vL,E[v|q]}$ equal $\alpha_{E[v|q],vH}$.\(^\text{14}\)

---

\(^{12}\) We assume this condition throughout the paper since otherwise informative advertising does not arise. The condition is $c < \frac{v_L(\sqrt{v_H^2 - 4v_L} - v_H - v_L + v_H^2 + v_H^2 - 5v_H + 3v_L^2)}{2(v_H - v_L)(v_H + v_L)^2}$.

\(^{13}\) Clearly, $\alpha_{i,j} = \alpha_{j,i}$. We omit argument $(q)$ for notational simplicity.

\(^{14}\) Solving for $\bar{q}$ yields two solutions, but one of them always falls outside the unit interval.
C Proof of Proposition 3 (Characterization of market outcome with known quality)

C.1 Search Probability

First, note that the search probability is strictly increasing in range \( q \in (q_0, \bar{q}) \):

\[
\frac{\partial \alpha^*_L (q)}{\partial q} = \frac{(v_H - v_L)(v_L + q^2 (v_H - v_L))}{(1-q)^2 (v_L + q (v_H - v_L))^2} > 0
\]  \( (35) \)

In the \( q \in (\bar{q}, q_1) \) the search probability is affected by product quality according to

\[
\frac{\partial \alpha^*_H (q)}{\partial q} = \frac{(v_H - v_L)(q^2 v_H - (1-q)^2 v_L)}{(v_L + (2-q)q (v_H - v_L))^2} \]  \( (36) \)

where its signal depends on the expression \( q^2 v_H - (1-q)^2 v_L \), with a unique solution in the unit interval at \( q' = \frac{\sqrt{v_L}}{\sqrt{v_H} + \sqrt{v_L}} \). It is easy to verify that \( \frac{\partial \alpha^*_H (q)}{\partial q} \) is decreasing for \( q < q' \), and increasing otherwise. Because \( q_1 > q' \), the slope of search is always increasing towards \( q_1 \). However, at \( q = \bar{q} \) the sign of \( q^2 v_H - (1-q)^2 v_L \) depends on the relation between \( v_H \) and \( 4v_L \). When \( 4v_L > v_H \), \( \frac{\partial \alpha^*_H (Q)}{\partial q} \big|_{q=\bar{q}} > 0 \), and when \( 4v_L < v_H \), \( \frac{\partial \alpha^*_H (Q)}{\partial q} \big|_{q=\bar{q}} < 0 \). The underlying intuition is explained in the text.

C.2 Expected Price

The expected price is given by

\[
E[p^* (q)] = \begin{cases} 
E[v | q], & q > q_1 \\
\beta^* (q) v_H + (1 - \beta^* (q)) E[v | q], & \bar{q} < q \leq q_1 \\
\beta^* (q) v_L + (1 - \beta^* (q)) E[v | q], & q_0 < q \leq \bar{q} \\
E[v | q], & q \leq q_0
\end{cases}
\]

It is easy to show that the average price is always monotonically increasing. When quality is very low \( (q \leq q_0) \) or very high \( (q > q_1) \) an increase in \( q \) translates to a higher willingness-to-pay by consumers who do not search, and the firm is able to charge more. In the intervals \( (q_0, \bar{q}) \) price is also increasing: Although \( \beta^* (q) \) is increasing in \( q \) near \( q_0 \) (which means that the firm...
is putting more weight on price \( v_L \) rather than on price \( E[v|q] \), the increase is less than the increase rate of \( E[v|q] \). To see this, note that

\[
\frac{\partial}{\partial q} (\beta^* (q) v_L + (1 - \beta^* (q)) E[v|q]) = \frac{c}{(1-q)^2} > 0
\]

On the next branch \((\bar{q}, q_1)\) the partial derivative of expected price w.r.t. quality yields

\[
\frac{\partial}{\partial q} (\beta^* (q) v_H + (1 - \beta^* (q)) E[v|q]) = \frac{c}{q^2} > 0
\]

which is also positive. Finally, inspection reveals that the expected price is continuous in \( q \), which finishes the proof (more precisely, the average price is continuous in the \( C_0 \) sense).

### C.3 Expected Firm Profit

The expected firm profit is given by

\[
E[\pi^*(q)] = \begin{cases} 
E[v|q], & q > q_1 \\
\alpha^*_H (q) q v_H, & \bar{q} < q \leq q_1 \\
v_L, & q_0 < q \leq \bar{q} \\
E[v|q], & q \leq q_0
\end{cases}
\]

It is easy to verify that expected profit is always increasing within each branch. For the branch \((\bar{q}, q_1)\) specifically the marginal effect of quality on expected profits is equal to

\[
\frac{\partial}{\partial q} (\alpha^*_H (q) q v_H) = \frac{v_H \left(2q^2 v_H^2 + (2-3q) q v_H v_L + (1-q)^2 v_L^2\right)}{((2-q) q (v_H - v_L) - v_L)^2} > 0
\]

The sign of the expression above can be found by showing that the numerator is positive at the endpoints of quality \( q \) (\( q = 0 \) and \( q = 1 \)) and that it is strictly increasing in \( q \):

\[
\text{Num} (\alpha^*_H (q) q v_H)|_{q=0} = v_H v_L^2
\]

\[
\text{Num} (\alpha^*_H (q) q v_H)|_{q=1} = v_H^2 (2v_H - v_L)
\]
\[
\frac{\partial}{\partial q} \text{Num} (\alpha^*_H (q) q v_H) = 2 v_H (v_H - v_L) (2 q v_H + (1 - q) v_L) > 0
\]

It remains to verify the expected profit at its discontinuities. Profit always decreases near the first jump (i.e., near \(q_0\)) because of the increased heterogeneity induced by search. When consumers start searching the firm can no longer charge price \(E[v|q]\) and serve the whole market. Instead, it introduces price \(v_L\) in order to target consumers who search and do not find a fit. Profit is continuous at \(\bar{q}\) and at \(q_1\) the difference in profit is always positive:

\[
E[v|q_1] - \alpha^*_H (q_1) q_1 v_H = \left( v_H - v_L - \sqrt{(v_H - v_L)(v_H - v_L - 4c)} \right) k
\]

where \(k > 0\).\(^{15}\) The expression is trivially positive for \(c > 0\). Finally, note that the difference in payoffs is well defined as long as \(4c \leq v_H - v_L\). This condition is equivalent to the parameter space allowing for search, i.e., \(q_0 < q_1\). This completes the analysis of expected profit with respect to quality.

### C.4 Consumer Surplus

Consumer surplus is equal to zero, except in region \(q \in (q_0, \bar{q})\). In this region it is equal to

\[
E(CS) = \beta (E[v|q] - v_L) = q(v_H - v_L) - \frac{c}{1-q}
\]

The results in Proposition 3 follow from taking partial derivates w.r.t. the variables of interest:

\(^{15}\)The constant \(k\) is equal to \(\frac{(v_H + v_L + \sqrt{(v_H - v_L)(v_H - v_L - 4c)})^2}{4(v_H - v_L)(2c + v_H + v_L + \sqrt{(v_H - v_L)(v_H - v_L - 4c)})}\).
\[ \frac{\partial}{\partial v_H} E(CS) = q > 0 \]
\[ \frac{\partial}{\partial v_L} E(CS) = -q < 0 \]
\[ \frac{\partial}{\partial c} E(CS) = -\frac{1}{1-q} < 0 \]
\[ \frac{\partial}{\partial q} E(CS) = v_H - v_L - \frac{c}{(1-q)^2} \]

The last derivative requires further inspection. First, evaluation at \( q_0 \) reveals a positive slope. The derivative has only one inflection point at \( q = 1 - \sqrt{\frac{c}{v_H-v_L}} \), and the second derivative is equal to \(-\frac{2c}{(1-q)^3}\), which reveals a strictly concave function. Evaluation at \( \tilde{q} \) reveals that the sign depends on how low the search cost is:
\[ \left. \frac{\partial}{\partial q} E(CS) \right|_{q=\tilde{q}} = v_H - v_L - \frac{c}{(1-\tilde{q})^2}, \]
which is positive as long as \( c \) is low enough. When \( c \) is high relative to the difference in valuations consumer surplus has an inverse-U shape.

## D Consumer Beliefs and Informativeness

It is worth specifying the set of beliefs consumers form throughout the game. Before search consumers form belief \( b_1(m) \) about the firm’s type and further believe \( b_{1}^{*}(m) \) to be the firm’s distribution of prices. Consumers who decide not to search still form beliefs about the firm’s type before the purchase decision, denoted as \( b_2(m, p) \). We define \( b_1(m) = \tilde{P}_r(H|m) \) and \( b_2(m, p) = \tilde{P}_r(H|m, p) \). Let \( m^* \) and \( p^* \) be the firm’s equilibrium message and price, respectively. Note that PBE requires \( b_1(m^*) = b_2(m^*, p^*) \) so that in equilibrium consumers cannot face an informative message that is later contradicted by price. Moreover, PBE does not speak to off-equilibrium path beliefs.

In the analysis we focus on cases in which price is not informative, i.e., \( b_2(m, p) = b_1(m), \forall p \), because as we show next, no separating equilibrium exists if price is informative. Consider the case in which \( b_2(m, p \in \{E[v|q_H], v_H\}) = 1, \forall m \) and \( b_2(m, p \in \{v_L, E[v|q_L]\}) = 0, \forall m \), such that consumers believe the firm is of high quality if \( p \in \{E[v|q_H], v_H\} \) and consumers believe the firm to be of low quality if \( p \in \{v_L, E[v|q_L]\} \). In this case the low-type firm would prefer to imitate the high-type by pricing at \( E[v|q_H] \), and earn \( E[v|q_H] (1 - \alpha + \alpha q_L) \) rather than
Because no single-crossing property precludes imitation. Finally, the assumption of uninformative prices is a common assumption in games with multiple signals (see Hart and Tirole, 1990 and Caminal and Vives, 1996 for examples).

D.1 Proof of Proposition 4 (Market outcome with unknown quality)

There exist 10 regions of interest. We first focus on the regions where one firm always has an incentive to deviate.

D.2 Region $q_L < q_H < q_0$

No search occurs in this region, and consumers are willing to pay $E[v|q_L]$ for a low quality product, and $E[v|q_H]$ for a high quality one. Given that no search occurs in this region, if believed the low type firm always has an incentive to imitate the high type and earn profits $E[v|q_H]$. Hence, no credible communication can take place.

D.3 Region $q_L < q_0 < q_H < q$ or $q_0 < q_L < q_H < q$

In the first sub-region the high type firm always wants to imitate the low type firm because it can increase its profits to $E[v|q_L]$ from $v_L$ if it is believed. In the second sub-region the high type firm also wants to imitate the low type firm because it can receive profit $E[v|q_L] \left(1 - (1 - q_H) \alpha^*_L(q_L)\right)$ rather than $v_L$ if believed. This is easily shown by inspection.

As for the low type firm, it never has an incentive to deviate. It earns profits $E[v|q_L]$ and $v_L$ in the first and second sub-regions, respectively, by advertising truthfully. It suffices to show that $v_L > \max \{E[v|q_H] \left(1 - (1 - q_L) \alpha^*_L(q_H)\right), \alpha^*_L(q_H) q_L v_H\}$ whenever $q_H < q$. The profit of the high type firm is useful: Because $E[v|q_H] \left(1 - (1 - q_H) \alpha^*_L(q_H)\right) = v_L$, it follows that $E[v|q_H] \left(1 - (1 - q_L) \alpha^*_L(q_H)\right) < v_L$. Also, we know that for $q_H < q$, $\alpha^*_L(q_H) q_H v_H < v_L$, so $\alpha^*_L(q_H) q_L v_H < v_L$ as well. Hence, in these regions only the high type firm has an incentive to imitate the low type one.
D.4 Region $q_1 < q_H$

This region is obtained by the union of sub-regions $q_L \in (0, q_0) \cup (q_0, \overline{q}) \cup (\overline{q}, q_1) \cup (q_1, 1)$, $q_H \in (q_1, 1)$. In all sub-regions the high type firm earns profit $E[v|q_H]$ when its type is revealed. In sub-region $q_L \in (0, q_0) \cup (q_0, \overline{q})$ it is clear that the low type firm has an incentive to claim it is high, because $E[v|q_H] > \{v_L, E[v|q_L]\}$. When $q_L \in (\overline{q}, q_1)$ the low type firm earns profit $E[v|q_L](1 - (1 - q_L)\alpha_H^*(q_L))$, which is also less than $E[v|q_H]$. Finally, when $(q_1, q_H)$ the low type firm earns $E[v|q_L]$, which again is lower than the profit it can earn if it successfully imitates the high type firm.

D.5 Region $\overline{q} < q_L < q_H < q_1$

In this region $\pi(L|\overline{L}) = \alpha_H^*(q_L) q_L v_H = E[v|q_L](1 - (1 - q_L)\alpha_H^*(q_L))$. If it successfully imitates the high type firm, the low type firm earns $\pi(L|\overline{H}) = \max\{\alpha_H^*(q_H) q_L v_H, E[v|q_H](1 - (1 - q_L)\alpha_H^*(q_H))\}$. Note that if the low type firm can induce more search through imitating the high type, it prefers to deviate because it earns $\alpha_H^*(q_H) q_L v_H > \alpha_H^*(q_L) q_L v_H$. Otherwise, it is clear that $E[v|q_H](1 - (1 - q_L)\alpha_H^*(q_H)) > E[v|q_H](1 - (1 - q_L)\alpha_H^*(q_L))$, and so the low type firm always prefers to deviate and imitate the high type firm if her communication is effective.

As for the high type firm, it if induces more search through imitating the low type firm then it can increase profits from $\alpha_H^*(q_H)q_H v_H$ to $\alpha_H^*(q_L)q_H v_H$. Otherwise - if $\alpha_H^*(q_L) < \alpha_H^*(q_H)$ - it always prefers to report truthfully because $E[v|q_H](1 - (1 - q_L)\alpha_H^*(q_H)) > E[v|q_L](1 - (1 - q_L)\alpha_H^*(q_L))$. The proof is simple but long.

In particular, calculate $E[v|q_H](1 - (1 - q_L)\alpha_H^*(q_H)) - E[v|q_L](1 - (1 - q_L)\alpha_H^*(q_L))$. The denominator becomes $(2 - q_L)q_L(v_H - v_L) + v_L$, which is always positive. Hence, the sign of the expression depends on the numerator, which is linear in $q_H$, and so the truth-telling arises when $A + B q_H > 0$. For truth-telling to emerge we require that $q_H > -\frac{A}{B}$ whenever $B > 0$ and that $q_H < -\frac{A}{B}$ whenever $B < 0$. It remains to verify that each of these conditions always holds. In particular, it is possible to verify that $q_L > -\frac{A}{B}$ holds whenever $B > 0$ (which in turn implies that $q_H > -\frac{A}{B}$). Moreover, when $B < 0$ it is also possible to verify that $1 < -\frac{A}{B}$, which

\[^{16}A = -\Delta^2(1 - q_L)q_H^2 + \Delta q_L v_L + v_L^2\] and \[B = 2\Delta^2(1 - q_L)q_L + v_L(\Delta - 2\Delta q_L) - v_l^2,\] where $\Delta \equiv v_H - v_L$. Moreover, $B \geq 0 \Leftrightarrow q_L \leq \frac{\sqrt{v_H(v_H - 2v_L) + v_H - 2v_L}}{2(v_H - v_L)}$, which is possible as long as $v_H > 2v_L$.  

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implies that $q_H < -\frac{4}{\Delta}$ always holds.

Hence, the low type firm always has an incentive to deviate from truth-telling. The high type firm may also want to deviate but only if it can induce more search when doing so.

D.6 Region $q_L < q_0 < \bar{q} < q_H < q_1$

In this case the low type firm earns $E[v|q_L]$ and the high type firm earns $\alpha_H^* (q_H) q_H v_H$ if they advertise truthfully. To understand the conditions under which credibility may arise, we first analyze the incentives of the low type firm.

- **Low type Firm:**

  If the low type firm deviates, it prefers to set price $p^* = E[v|q_H]$ rather than $p^* = v_H$. This can be shown by comparing profits

  $$\pi \left( L|H \right) \bigg|_{p^* = E[v|q_H]} \geq \pi \left( L|H \right) \bigg|_{p^* = v_H}$$

  $$\Leftrightarrow E[v|q_H] (1 - (1 - q_L) \alpha_H^* (q_H)) \geq \alpha_H^* (q_H) q_L v_H$$

  $$\Leftrightarrow 1 - (1 - q_L) \alpha_H^* (q_H) \geq \frac{q_L v_H}{q_H v_H + (1 - q_H) E[v|q_H]}$$

  $$\Leftrightarrow 1$$

  which holds because $q_H > q_L$ and $v_H > E[v|q_H]$.

  It remains to show under which conditions the low type firm is better off advertising truthfully, i.e., when

  $$\pi \left( L|L \right) > \pi \left( L|H \right) \bigg|_{p^* = E[v|q_H]}$$

  $$\Leftrightarrow E[v|q_L] > E[v|q_H] (1 - (1 - q_L) \alpha_H^* (q_H))$$

  The tradeoff for the low type firm is that by overstating its quality it receives a higher price but sells less units. Determining the truth-telling condition is simple but again, a relatively long process. The denominator of the difference in profits is equal to $(2 - q_H) q_H \Delta + v_L$, which is always positive ($\Delta \equiv v_H - v_L$). The firm prefers to advertise truthfully when $A - B q_L > 0$.\footnote{\[ A \equiv -\Delta^2 (1 - q_H) q_H^2 + \Delta q_H v_L + v_L^2 \text{ and } B \equiv -2\Delta^2 (1 - q_H) q_H - v_L \Delta (1 - 2q_H) + v_L^2. \]}
follows that $A \geq 0 \Leftrightarrow \frac{\Delta}{v_L} \leq \frac{1+\sqrt{4A-4B}}{2(1-B)}$ and $B \geq 0 \Leftrightarrow \frac{\Delta}{v_L} \leq \frac{-1+2\sqrt{4A+4B}}{4(1-A)}$. When $A, B > 0$, $\frac{A}{B} > 1$, and so truth-telling is always satisfied (we show this below). When $A < 0 \Rightarrow B < 0$ we require that $q_L > \frac{A}{B}$, which trivially holds since $\frac{A}{B} < 0$. Finally, the condition $A > 0 \land B < 0$ translates into $\frac{\Delta}{v_L} > \frac{1+\sqrt{4A-4B}}{2(1-B)}$ and $q_L > \frac{A}{B}$.

To verify that $\frac{A}{B} > 1$ whenever $A, B > 0$, first notice that $\frac{-1+2\sqrt{4A+4B}}{4(1-A)} < \frac{1+\sqrt{4A-4B}}{2(1-B)}$, so we require $\frac{\Delta}{v_L} > \frac{1+\sqrt{4A-4B}}{2(1-B)}$. Moreover, $\frac{\partial}{\partial \Delta} \frac{A}{B} > 0$ and $\min \Delta \frac{A}{B} = 1$ at, $\Delta = 0$. Hence, it follows that $\frac{A}{B} > 1$.

In summary, the low type firm prefers to tell the truth if $A > 0$, or if $A < 0$ and $q_L > \frac{A}{B}$. We label this condition set as $C_1$: $\frac{\Delta}{v_L} \leq \frac{1+\sqrt{4A-4B}}{2(1-B)} \vee \left(\frac{\Delta}{v_L} > \frac{1+\sqrt{4A-4B}}{2(1-B)} \land q_L > \frac{-\Delta^2(1-q_H)q_L^2+\Delta q_L v_L + v_L^2}{2\Delta^2(1-q_H)q_L v_L + \Delta q_L v_L + v_L^2}\right)$.

• High type Firm:

The high type firm may also benefit from imitating the low type firm and receive $E[v|q_L]$. It is better off advertising truthfully when $\alpha_H^*(q_H)q_Hv_H \geq E[v|q_L]$, which solving w.r.t. $q_H$ yields

$$q_H \geq \frac{2\Delta^2 q_L + \Delta v_L - v_L^2 + \sqrt{4\Delta^4 q_L^2 + 4\Delta^3 q_L^2 v_L + 8\Delta^2 q_L v_L^2 + 8\Delta^3 q_L v_L + 6\Delta v_L^2 + 5\Delta^2 v_L^2 + v_L^4}}{2\Delta(\Delta + \Delta q_L + 2v_L)},$$

where $\Delta \equiv v_H - v_L$. This completes the set of conditions $C_1$.

Finally, note that it cannot happen that both firms want to deviate. When the low type firm prefers to overstate its quality, condition $E[v|q_L] < E[v|q_H]$ $(1 - (1-q_L)\alpha_H^*(q_H))$ holds. However, this implies that $E[v|q_L] < E[v|q_H]$ $(1 - (1-q_H)\alpha_H^*(q_H))$, which is the condition that ensures truth-telling for the high type firm.

We summarize the conditions for credible communication at the end of the next section.

**D.7 Region $q_0 < q_L < \bar{q} < q_H < q_1$**

In this region credibility may also arise, and the method of proof is similar to that of the previous section.

• Low type Firm
First, note that if the low type firm deviates, it does so using price $p^* = E[v|q_H]$. The low type firm is better off advertising truthfully as long as its quality is low enough, i.e.,

$$\pi(L|\hat{L}) \geq \pi(L|\hat{H})|_{p^* = E[v|q_H]} \iff v_L \geq E[v|q_H](1 - (1 - q_L)\alpha_H^*(q_H))$$

$$\iff q_L \leq 1 - \frac{1}{\alpha_H^*(q_H)}\left(1 - \frac{v_L}{E[v|q_H]}\right).$$

The reason is that when its quality is low, the low type firm loses demand if it imitates the high type firm, in particular of consumers who search but do not find a fit. We denote this condition as $C_2$, applicable to the low type firm.

- **High type Firm**

The high type firm would prefer to understate its quality if this results in higher search. In this case it can charge $p^* = v_H$ and earn profit $\alpha_L^*(q_L)q_Hv_H$ rather than $\alpha_H^*(q_H)q_Hv_H$. However, when consumers search the low type firm less than the high type one ($\alpha_L^*(q_L) < \alpha_H^*(q_H)$) the firm may have an incentive to advertise truthfully. In case of deviation its best option is to charge $p^* = E[v|q_L]$. Solving inequality $E[v|q_H](1 - (1 - q_H)\alpha_H^*(q_H)) > E[v|q_L](1 - (1 - q_H)\alpha_L^*(q_L))$ yields

$$\Delta \frac{v_L}{v_H} > q_H(1 - q_H) - q_L^2 + \sqrt{q_H^2(1 - q_H)(1 - 5q_L) + 2q_H(2 - q_L^2 - q_H)q_L + q_L}$$

$$\frac{2q_Hq_L(1 - q_H + q_L)}{2q_Hq_L(1 - q_H + q_L)}.$$ 

The threshold is increasing in $q_H$, which means that the incentive for the high type to deviate increases with $q_H$. This completes the characterization of the region $C_2$, required for advertising credibility to emerge. As in the previous case firms never have a simultaneous incentive to deviate.

Hence, credible communicate takes place if and only if

$$C_0 \equiv (q_L < q_0 < q_H < q_1 \land C_1) \implies (q_0 < q_L < q_H < q_1 \land C_2).$$ 

Existence can be verified by inspection: In region $q_L < q_0 < q_H < q_1$ (first term of region $C_0$) it is easy to verify that all credibility conditions are satisfied at $v_H = 1, v_L = \frac{1}{16}, c = \frac{1}{16}, q_H^{23} = \frac{125}{128}, q_L = \frac{5}{572}$. In region $q_0 < q_L < q_H < q_1$ it is easy to verify that credibility is satisfied at
\[ v_H = 1, v_L = \frac{1}{10}, c = \frac{5}{1024}, q_H = \frac{157}{1024}, q_L = \frac{3}{256}. \]

E Proof of Proposition 5 (Characterization of market outcome with informative advertising)

We have already shown that expected price is strictly increasing in quality for the case of perfect information. Since the informative advertising case constitutes a linear combination of scenarios of the known quality case, it follows that ex-ante price under informative advertising is also strictly increasing in quality levels.

Expected profits depend on the particular region of interest. When \( q_L < q_0 \) the low type firm earns profits \( E[v \mid q_L] \) and the resulting ex-ante profit becomes

\[
E(\pi \mid q_L < q_0) = (1 - \lambda) E[v \mid q_L] + \lambda \alpha^*_H (q_H) q_H v_H
\]

When \( q_L > q_0 \) ex-ante profit is equal to

\[
E(\pi \mid q_L > q_0) = (1 - \lambda) v_L + \lambda \alpha^*_H (q_H) q_H v_H
\]

In both cases profits are strictly increasing in the quality levels. However, at \( q_L = q_0 \) expected profits decrease due to the search regime starting for the low type firm. The statement about consumer surplus trivially follows from Proposition 3.
References


